

Consumer Price Indices Consultation

1. INTRODUCTION

A typical household would budget its expenditure by adding the total costs of the contents of a typical basket of goods. To work out the average cost of their purchases they would divide the total cost by the number of items. This gives what is known as the Arithmetic Mean. Should they wish to work out their annual inflation rate they would sum the cost of the items in their basket at the start of the year and then at the end of the year and divide the difference by the cost at the start of the year. That is the same as the difference between the two arithmetic means divided by the start of the year's arithmetic mean. The typical household would conduct all of its budgeting on the basis of the total costs of its basket. This principle is well understood.

The inflation value so obtained meets the requirements of the Social Administration Act 1992 Para 150 (2) that states:

(2) Where it appears to the Secretary of State that the general level of prices is greater at the end of the period under review than it was at the beginning of that period, he shall lay before Parliament the draft of an up-rating order-
(a) which increases each of the sums to which subsection (3) below applies by a percentage not less than the percentage by which the general level of prices is greater at the end of the period than it was at the beginning.

It is generally believed that the percentage by which the general level of prices is greater at the end than it was at the beginning was intended to be reflected by the change in the total costs, weighted by the sales, of the contents of a typical basket purchased by a typical household.

2. CRITERION

An index used to uprate pensions and benefits should reflect the changes in total expenditure of these groups. Hence the contents of the typical basket should reflect the typical expenditure of the typical household. The index chosen should therefore be an Uprating Index, or as described by John Astin and Jill Leyland a Household Inflation Index¹. The mathematics used should also reflect the percentage increase in the total weighted cost of the typical basket chosen. It is not important as to whether such an index is of advantage to the benefiting groups or not only that it is correct, understandable and justifiable.

¹ Astin, J. & Leyland, J: "Towards a Household Inflation Index", (RSS) May 2015.

3. CALCULATION METHODS

There are three mathematical methods used in the currently produced Price Indices, Dutot, Carli and Jevons. These should be analysed to see if they meet the criterion as spelt out in section 2.

3.1 Dutot

The increase in the total weighted costs cost at the end of the accounting period multiplied by 100 and divided by the original total weighted cost will give the percentage inflation index as understood by most people. This can be obtained by the Dutot formula, which is:

$$\frac{\sum_{p=1}^{p=n} b_p}{\sum_{p=1}^{p=n} a_p}$$

Where the basket of goods contains 'n' items, each item 'p' costing 'a_p' at the start of the period and 'b_p' at the end. For most of us this is easy to understand and to justify.

$$\frac{\sum_{p=1}^{p=n} b_p}{\sum_{p=1}^{p=n} a_p} = 1 + \frac{\sum_{p=1}^{p=n} b_p - \sum_{p=1}^{p=n} a_p}{\sum_{p=1}^{p=n} a_p} = 1 + \frac{\text{Increase in total weighted cost of the basket}}{\text{Total weighted cost at start of accounting period}}$$

The index so obtained is related to the total cost of the basket and is both correct and justifiable and hence, meets the criterion laid out in section 2.

3.2 Carli and Jevons

Carli and Jevons' suitability for calculating inflation indexes depend on the same fact that they both entirely rely on the same fractions for all their items in the basket. Once the true meaning of their formula is established the problems that arise from using these fractions can be considered with regard to both Carli and Jevons at the same time.

3.2.1 Breakdown of the Formulae used to Calculate Carli and Jevons

In both cases consider a basket of goods containing 'n' items, item 'p' costing 'a_p' at the start of the period and 'b_p' at the end.

3.2.1.1 Carli

The Carli method takes the mean of the ratios of the cost of each item at the end and the start of the period in question, that is the Average of Price Relatives and is given by:

$$\frac{\sum_{p=1}^{p=n} \frac{b_p}{a_p}}{n} = \frac{1}{n} \sum_{p=1}^{p=n} (1 + \Psi_p) \quad \text{where } \Psi_p \text{ is uprating ratio of the 'p'}^{th} \text{ item}$$

The Carli gives the mean of the individual uprating rates but does not consider the individual contributions to the basket.

3.2.1.1 Jevons

The Jevons method uses the ratio of the Geometric Mean and is the nth root of the products of the ratio of the weighted costs of the items at the end and the start of the period in question. The value given by Jevons is:

$$\sqrt[n]{\frac{b_1 \times b_2 \times b_3 \dots b_{(n-1)} \times b_n}{a_1 \times a_2 \times a_3 \dots a_{(n-1)} \times a_n}}$$

That is:

$$\sqrt[n]{\prod_{p=1}^{p=n} \frac{b_p}{a_p}} = \sqrt[n]{\prod_{p=1}^{p=n} (1 + \Psi_p)} \quad \text{where } \Psi_p \text{ is uprating ratio of the 'p'}^{th} \text{ item}$$

The Jevons only uses the individual uprating rates but does not consider the individual contributions to the total value of the basket and only gives the geometric mean of the individual uprating rates but does not consider the individual contributions to the basket.

3.2.2 Consideration of the Use of Only Uprating Ratios

It should be noted that both Carli and Jevons only use uprating ratios of the individual items and do not use the total weighted costs of the items in the basket.

The effect is to give a mean of the uprating factors and not the ratio of the total cost of the basket which is what would give the true inflation rate. If the uprating factor is the same for two wildly differing costed items then they are treated exactly the same as each other. However the more expensive item will have a greater influence on the total cost of the basket and hence using the ratio does not reflect the inflation index of the actual total cost of the basket at the end and the start of the accounting period.

Using the uprating ratio for each item means that neither the Carli or the Jevons has any relation to the inflation experience of the basket. Hence with both Carli and Jevons it is impossible to obtain the actual inflation index of the total cost of the basket except by accident. It should also be noted that provided there is no significant change in weighting over the accounting period there is no point in calculating the weighting as it will be cancelled out when dividing the weighted costs at the end by the weighted costs at the start.

4. ARGUMENTS AGAINST DUTOT

Two arguments are levelled at Dutot regarding its use in evaluating Price Indices. First that it does not reflect changes in purchasing practice and second that it does not deal with the belief that where a category of goods contains low volume high cost items along with high volume low cost items the high cost item could have a disproportionate effect on the Price Index.

4.1 Changes in Purchasing Practice

The first problem arises from the assumption that when money is scarce there will always be a change in purchasing practice towards cheaper items. The example used is that if there were associated items such as Kellogg's Cornflakes and Tesco's Cornflakes purchasers will move from the dearer item to the cheaper one.

To be justified the method chosen to reflect changes in purchasing practice should show this change reducing as the opportunity for such changes are exhausted. Jevons fails to do this and makes a similar correction irrespective of the change in purchasing practice. The use of Jevons does not therefore reflect the magnitude of the weighted costs and the resultant index is not based on the data's underlying evidence.

I would have expected that where this change in purchasing practice occurs it should have already been reflected by the changes in the weighted arithmetic mean, which is managed on the basis of what did happen in the previous year. If this is not the case the problem needs to be examined further.

Consider the feasibility of treating associated items separately. By treating those associated items where changes in purchasing practice could occur as separate items the resultant cost of living indices would be

seen to be based on the actual evidence. Such an approach would illustrate the actual change in purchasing practice including the times when these changes would start to run out.

If this task, although ideal, is too great for the ONS they must have some knowledge of the proportion of items that may be subject to changes in purchasing practice and would have a significant effect. A selection of a few of these items could be made where they would reflect the change in purchasing practice of the whole group which may be subject to change. Such an approach would at least be evidence based and would reflect any tail-off in the change in purchasing practice.

It should be noted that the selection of a few items is similar to selecting a typical basket of goods to evaluate the cost of living index instead of taking all purchases into account.

4.2 Wide Variations in Item Costs and Volumes of Sales

The second problem involves the belief that in those categories of goods that contain low volume high cost items in the same category as high volume low cost items the high cost item could have a disproportionate effect on the Price Index. The example often used is the effect grand pianos could have in the musical instrument category when compared to recorders.

Data extremes are not new to the calculation of Price Indices. The Grand Piano problem is typical of a situation that has faced Price Indices since their conception. The original problem is that of the population used to obtain the data and the excessive influence the better off and the poorest can have on the index when it is compared to the inflation experienced by the typical household.

A recent pioneering paper by Flower and Wales² concluded that the CPI is broadly representative of the price experience of households around two-thirds of the way up the expenditure distribution. They also concluded that it was representative of the experience of households nine-tenths up the income distribution. Whatever method is used to obtain a typical household index, in spite of the richest distorting the application of the index, must be able to deal with the Grand Piano problem.

Computing capacity has continuously increased over the years and both these problems could now be readily dealt with by using this increased capacity.

The use of price ratios instead of prices in the RPI and CPI does eliminate this problem but is not evidence based and does not take account of the contribution of these items to the general level of prices in the basket. It is not logical to attempt to solve one problem by adopting an irrational solution.

5 CONCLUSION

Dutot is widely recognised, understood and accepted as the appropriate method to obtain inflation indices. The situations that could arise and which put the use of Dutot in doubt can all be tackled even when still using Dutot by methods that are sketched out in the enclosed paper³. These solutions should be considered along with others but not Carli or Jevons that are clearly flawed and illogical. It should be noted that with regard to the Grand Piano, and possibly other goods, their problem may well be self-correcting if trimming of the richest and poorest is adopted for the population. As we now have the computing power to calculate correct indices and deal with exceptional circumstances as they arrive we should not resort to methods such as Carli and Jevons that are not logically supportable.

² Flower, T. & Wales, P: "Variations in the inflation experiences of UK households", ONS. (2014)

³ Dean, D. J: "Dutot - Enabling its use in all Price Indices", (Thames Valley Pensioners Convention), Unpublished Paper Enclosed

DUTOT - ENABLING ITS USE IN ALL PRICE INDICES

1. BACKGROUND

J. Astin¹ has argued that a Household Inflation Index - particularly if it is to be the main headline inflation index - should be recognisable and understandable in general terms to the "man in the street".

A typical household would budget its expenditure by adding the total costs of the contents of a typical basket of goods. To work out the average cost of their purchases they would divide the total cost by the number of items. This gives what is known as the Arithmetic Mean. Should they wish to work out their annual inflation rate they would sum the cost of the items in their basket at the start of the year and then at the end of the year and divide the difference by the cost at the start of the year. That is the difference between the arithmetic means divided by the first year's arithmetic mean. The typical household would conduct all of its budgeting on the basis of the total costs of its basket. This principle is well understood.

Mathematically the ratio of the total costs at the start of the period and at the end is called the Dutot, developed in 1738 and illustrated by:

$$\frac{\sum_{p=1}^{p=n} b_p}{\sum_{p=1}^{p=n} a_p}$$

Where the basket of goods contains 'n' items, each item 'p' costing 'a_p' at the start of the period and 'b_p' at the end. For most of us this is easy to understand and to justify.

However two arguments are levelled at Dutot for its use in evaluating Price Indices. First that it does not reflect changes in purchasing practice and second that it does not deal with the belief that where a category of goods contains low volume high cost items along with high volume low cost items the high cost item could have a disproportionate effect on the Price Index.

In the current indices these problems have been dealt with by the use of Carli and Jevons. Both these mathematical techniques solve the problems by cancelling out the magnitude of the weighted cost of the individual items at the start of the evaluation and at the end and then just using the uprating ratio of each item. In both Carli and Jevons the actual weighted costs are not used, only the uprating ratio, thus cancelling out the magnitude of the cost of the item. Hence, the Carli and the Jevons only involve the independent inflation rates of each particular item. Because they do not calculate the total cost of the basket of goods they can be shown to give inflation rates that do not reflect the changes in the cost of the basket².

If one item is more expensive than another its inflation rate carries no more influence on the Price Index when calculated by Carli or Jevons yet the more expensive item will clearly have a greater effect on the increased cost of the basket if the weighting is the same. With Carli or Jevons the magnitude of the weighted cost is irrelevant to the resulting Price Index.

¹ Astin. J: "Options for a Household Inflation Index", (UK Statistics Authority), 2014

² Dean. D. J: "The Mathematics of Price Indices", (Thames Valley Pensioners Convention), 5 April 2015.

Both Carli and Jevons are blanket solutions to the claimed weaknesses in the use of Dutot. However, they deal with the problems by failing to evaluate the change in the total cost of the basket or its inflation rate. They just cancel out the costs at the start and end giving an inflation rate for each item separately. They do in effect ignore the changes in the cost of the basket and are not therefore evidence based and the resulting index is arbitrary.

2. SOLUTIONS TO BOTH PROBLEMS USING DUTOT

Computing capacity has continuously increased over the years and both the problems could now be readily dealt with by using this increased capacity.

2.1 Dutot and Changes in Purchasing Practice

The first problem arises from the assumption that when money is scarce there will always be a change in purchasing practice towards cheaper items. The example then used was if there were associated items such as Kellogg's Cornflakes and Tesco's Cornflakes purchasers will move from the dearer item to the cheaper one.

I would have expected that where this change in purchasing practice occurs it should have already been reflected by the changes in the weighted arithmetic mean, which is managed on the basis of what did happen in the previous year. If this is not the case the problem needs to be examined further.

To be justified the method chosen to reflect changes in purchasing practice should show this change reducing as the opportunity for such changes are exhausted. Jevons fails to do this and makes a similar correction irrespective of the change in purchasing practice. The use of Jevons does not therefore reflect the magnitude of the weighted costs and the resultant index is not based on the data's underlying evidence.

The use of Jevons makes the assumption that purchasing practice will continue to change in the same proportion in the subsequent years irrespective of whether the change had already occurred in the previous years and there are no other cheaper items available. There is also a measure of circularity in these arguments; if the purchasing power of one's income is reduced on the assumption that the purchaser has bought the cheaper item then they may be forced to buy the cheaper item. The use of Jevons deals with the Cornflakes problem by causing an arbitrary reduction in the Prices Index for everything. Its use assumes there will always be a change in purchasing practice towards cheaper items and the change will continue even after all purchases are of the cheapest items. In other words even after it becomes impossible to buy cheaper items the cost of living index will assume cheaper items are still being bought. The reduction in the cost of living index when based on the geometric mean is arbitrary and not based on any actual evidence.

2.1.1 Sub Grouping in a Major Category

Let us consider the feasibility of treating associated items separately. By treating those associated items where changes in purchasing practice could occur as separate items the resultant cost of living indices would be seen to be based on the actual evidence. Such an approach would illustrate the actual change in purchasing practice including the times when these changes would start to run out.

If this task, although ideal, is too great for the ONS they must have some knowledge of the proportion of items that may be subject to changes in purchasing practice and would have a significant effect. A selection of a few of these items could be made where they would reflect the change in purchasing

practice of the whole group which may be subject to change. Such an approach would at least be evidence based and would reflect any tail-off in the change in purchasing practice.

It should be noted that the selection of a few items is similar to selecting a typical basket of goods to evaluate the cost of living index instead of taking all purchases into account.

2.1,1,1 TYPICAL EXAMPLE

Consider the basket of goods used in the second example in reference 2, that is where the initial values of the items were 1, 2, 3, 4, 5, and 6. Assume that all of the items are given a weight of 'w'. Also assume that item 3 is Cornflakes. Number of people buying cornflakes is 'w' and the average cost of cornflakes is 3.

Let the cost of Kellogg's Cornflakes be 3.1 and the Cost of Tesco's Cornflakes 2.6

Let the number of People who buy Kellogg's be $0.8w$ and who buy Tesco's $0.2w$

Total cost of Cornflakes in the basket is $3w$.

Let $0.1w$ people stop buying Kellogg's and start buying Tesco's

Total Cost of Cornflakes is $2.95w$.

If a further $0.1w$ stop buying Kellogg's and start to buy Tesco's, then the total cost of Cornflakes would reduce to $2.9w$.

By separating Cornflakes into two items the whole analysis would then be evidence based while at the same time reflecting changes in purchasing practice. The technique will also show a reduction in the effect as there are less people available in the position where they are able to change their purchasing practice.

2.2 Dutot and Wide Variations in Item Costs and Volumes of Sales

The second problem involves the belief that in those categories of goods that contain low volume high cost items in the same category as high volume low cost items the high cost item could have a disproportionate effect on the Price Index. The example often used is the effect grand pianos could have in the musical instrument category when compared to recorders. The use of price ratios instead of prices in the RPI and CPI does eliminate this problem but is not evidence based and does not take account of the contribution of these items to the general level of prices in the basket.

2.2.1 Trimming Solution

Data extremes is not new to the calculation of Price Indices. The Grand Piano problem is typical of a situation that has faced Price Indices since their conception. The original problem is that of the population used to obtain the data and the excessive influence the better off and the poorest can have on the index when it is compared to the inflation experienced by the typical household.

An index, such as the CPI, which gives equal weight to the expenditure of the whole population, is unlikely to reflect the expenditure levels and consumption patterns of the typical household. In fact, a recent pioneering paper by Flower and Wales³ concluded that the CPI is broadly representative of the price experience of households around two-thirds of the way up the expenditure distribution. The

³ Flower, T. & Wales, P: "Variations in the inflation experiences of UK households", ONS. (2014)

devisors of RPI recognised this problem and tackled it by removing from the weighting the expenditure of the extreme ends of the population distribution (a variation of a general method known as "trimming"). To be precise, the expenditures of the highest-income 4% of the households and also that of pensioner households which derive a least three quarters of their total income from state pensions and benefits are excluded from RPI coverage.

The result of these exclusions is an index which could be said to approximated, in a somewhat rough-and-ready way, that of a household-weighted index. This is because trimming removes the influence of such extreme values from the mean value, rendering the latter more representative of the distribution. There is a reason for this practice for the RPI. The RPI was launched in the later 1940s and the 1950s as a successor inflation index to the earlier cost of living index, founded in 1914. The latter was designed as a subsistence index, used to compensate for increases in the basic household costs of workers. It was thus logical that it excluded both richer households and those who no longer worked and were dependent on the state.

Although the removal of the highest 4% income and those pensioner households who derive at least 3/4 of their income from state pensions is arbitrary the principle is logical and could clearly be applied to the Grand Piano problem. The magnitude of the problem would have to be properly assessed and the amount of trimming then selected, but by doing that the method would be evidence based and therefore acceptable to the wider population.

2.2.2 Subdivision Solution

Because computing capacity has continuously increased over the years it would be easy to subdivide any category that was due to cause problems. Hence the musical instruments category could be subdivided into cheap high sales sub group, medium price medium sales sub group and a high price low sales sub group. Each sub group would be weighted in accordance with its contribution to the Household Budget. Such a grouping would clearly be accepted as being evidence based.

2.2.3 Multi Category Solution

Another solution where there are wide variations in costs and sales is by modifying the weighting of all those categories where the spread of weighted costs could possibly cause problems. A category could be selected where the effect of the problem was middle of the road for all the categories with the same problem. A required correction to the weighting could then be obtained and then applied to all relevant categories. This is no different from the use of a typical basket of goods to represent the total expenditure of all households.

3 CONCLUSION

Dutot is widely recognised, understood and accepted as the appropriate method to obtain inflation indices. The situations that could arise which put the use of Dutot in doubt can all be tackled even when still using Dutot by the methods described in this paper. These solutions should be considered along with others but not Carli or Jevons that are clearly flawed and illogical. It should be noted that with regard to the Grand Piano, and possibly other goods, their problem may well be self-correcting if trimming of the richest and poorest is adopted for the population.

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