# Consumer Price Indices in the UK

## Mark M Courtney

## Summary

This paper assesses the array of official inflation indices in terms of their suitability as an uprating index that measures the purchasing power of wages and pensions. The main findings are set out below:

- Overall, taking account of both coverage and formula effect differences and within the limitations of how price data is collected within the UK, the Retail Prices Index (RPI) is as good a consumer price index as one can get for uprating purposes. The systemic difference between the RPI and the Consumer Prices Index (CPI), currently (2015) running at 1.0 percentage points per annum, results predominantly from under-estimation by the CPI.

- The coverage of the RPI is targeted on the working population, as it excludes pensioners, tourists and the wealthiest 4% of households. In contrast to CPI, it also includes the owner occupier housing costs that form a major element of most household expenditure. These differences in coverage have caused the CPI to be lower than the RPI by an average of 0.3 percentage points per annum over the seventeen years since the CPI was introduced.

- The RPI and CPI use different statistical methods (known as aggregation formulae) for calculating average price changes for many of the items covered. The RPI uses an arithmetic average, usually in the form of the Average of Relatives, whereas the CPI, following the requirements of the EU's Harmonised Index of Consumer Prices, uses the Geometric Mean. Because the UK has unusually broad definitions of its items, with consequent variations of inflation within items, this formula effect difference is unusually large in the UK: it has averaged 0.6 percentage points a year since the introduction of the CPI (but 0.9 percentage points over the last five years, following a change in price-collection methodology). The choice of elementary aggregation formula is therefore the most important difference between the RPI and CPI.

- The "stochastic approach" used by statisticians as a method of assessing price indices shows that the Average of Relatives formula used in the RPI is an unbiased estimator of the average rate of inflation (defined as a Laspeyres-type index, which is the index used by all consumer price indices in the world for calculating the overall rate of inflation), whereas the Geometric Mean used in the CPI is biased downwards. This implies that the formula effect represents an additional under-estimate of inflation by the CPI by, on average, 0.6 percentage points.

- The whole formula-effect under-estimate when using the Geometric Mean has been defended in the past on the grounds that it is allowing for consumer substitution towards goods whose prices have risen more slowly. But this neglects the fact that price changes are driven by changes in both supply and demand, not in supply alone, so that trying to allow for consumer substitution does not favour one aggregation formula over another.

The ONS now accepts that this Economic argument is no reason to modify the stochastic approach.

- If the aim, unusually, is to estimate not a Laspeyres, fixed-basket price index but one, such as the Fisher Index, which includes the effect of quantity changes, the stochastic approach indicates that the relative performance of the Average of Relatives and the Geometric Mean depends on the data – how strongly consumers and suppliers react to changes in relative prices and whether demand or supply changes predominate.

- The theoretical results of the stochastic approach, both for the Laspeyres price index and for the Fisher Index have been supported by empirical research published by the Office for National Statistics.

- It is sometimes argued that the process of re-weighting the RPI and CPI every year to allow for changing patterns of expenditure will introduce an upward bias, due to "price-bouncing," in the RPI. However, there is no theoretical reason why any bias should be upwards and the available empirical evidence is that any such effect is very small – much less than 0.1 percentage points per annum.

- Although the Average of Relatives is widely used internationally in the calculation of other price indices, such as producer price indices or trade price indices, the UK is almost alone in using it in a consumer price index, and it has been alleged that it is therefore not consistent with best international practice. In fact, few countries have ever used the Average of Relatives for their consumer price index, since many of them have tightly defined, homogeneous items, unlike the UK, and use the Ratio of Averages (a different formulation of the arithmetic average) throughout. Fifteen years ago Australia and the USA switched their consumer price indices from using the Average of Relatives to the Geometric Mean and their example might be relevant, except that their reason for switching was to account for consumer substitution, and the ONS now considers that the Economic approach that gave rise to the idea of consumer substitution does not provide even a weak reason for such a switch.

- In March 2013, the UK Statistics Authority downgraded the classification of the RPI from National Statistic to official statistic. It made clear that it was doing so solely on the basis of evidence presented by the ONS, without following its usual practice of written consultation or discussion with outside experts, and the substantive reason for its decision was that the National Statistician had decided that the ONS would henceforth perform only routine updating of the RPI, which violated the UK Statistics Authority's requirement for continuous improvement. In the circumstances, the withdrawal of National Statistic designation from the RPI lacked a convincing statistical basis and cannot be regarded as a comment on its current accuracy.

- Attempts have been made to add owner-occupier housing expenditure to the CPI by introducing a new, experimental price index, CPIH. This uses a rental equivalence measure for housing costs which is a very poor proxy for expenditure that home owners actually incur and has been accompanied by data problems. Consequently, CPIH has so far gained little acceptance.

# Consumer Price Indices in the UK

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## 1. Introduction

It is generally accepted that there are advantages to having more than one consumer price index, in view of the different uses to which such an index can be put.[1] The two most important uses are as an indicator for monetary policy and as an uprating index, to measure the purchasing power of wages and pensions. Although we shall make reference to the properties of a good monetary policy indicator, our main concern in this note will be with the second type of index. Historically and often in general usage this is referred to as a cost-of-living index, but that term has become ambiguous since some statisticians use it to refer specifically to a constant-utility index (defined as a price index such that if incomes are increased in line with it, consumers' welfare remains unchanged). We shall, therefore refer to the second type of index simply as an uprating index.

Even though there is scope for more than one consumer price index, the UK has ended up with a confusingly large number – nine at the last count that have some claim to apply to the population at large, as well as two or three more that aim to measure the prices faced by sub-groups within the population. Towards the end of this note, we shall give brief descriptions of them all, but our main focus will be on just the two most prominent ones, from one or other of which all the rest derive. These are the Retail Prices Index (RPI), which was developed as an uprating index but, in a slightly modified form, was also used as a monetary policy indicator from 1975 until 2003; and the Consumer Prices Index (CPI), which was developed as a monetary policy index but has also been used as an uprating index since 2011.

We shall start with a brief description of the RPI and CPI and then move on to an assessment of how they perform as uprating indices. Since the CPI systematically shows a lower rate of inflation than the RPI – typically around one percentage point per annum lower – its introduction for uprating purposes has been controversial. With such a significant difference, it cannot be the case that they are equally valid measures of the purchasing power of money. Either one or the other must be correct, or the truth must lie somewhere between the two.

With so much at stake, there has been a flurry of analysis and research into the RPI and CPI in recent years, and consequently our assessment of them has to be quite detailed. Apart from passing references to two or three formulae, we have used non-mathematical language throughout, but there are footnote references to the original research for those who wish to look up more rigorous mathematical expositions of the same arguments. We have also included in Appendices extracts or summaries of five of the more recent research results.

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[1] See the discussion in J. Leyland, *What should an uprating consumer price index look like?*, published on Statsusernet, January 2014 at
http://www.statsusernet.org.uk/communities/resources/viewdocument/?DocumentKey=0cb5a678-9e41-4a9c-aa39-b35aed53bf7f&MessageKey=b1396916-017c-4450-8146-06d872670c62 .
The suggestions in this paper are developed in the paper by J. Astin and J. Leyland, *Towards a Household Inflation Index*, published on Statsusernet, May 2015 at
http://www.statsusernet.org.uk/viewdocument/?DocumentKey=e4f517b3-f381-417b-8233-fdce26f40f20

## 2. The Retail Prices Index

*Background: The Development of the Retail Prices Index*

The first systematic cost-of-living index in Britain was compiled and published monthly by the Board Of Trade from July 1914. This calculated the cost of a fixed basket of goods, with weights based on 1914 working class household expenditure and with a monthly inquiry into retail prices. The index covered food, housing, clothing, fuel and light, with only a 4% weight given to "other" items.[2] This cost of living index was used during 1914-1918 as the basis for wage increases to compensate for wartime price increases. It continued to be calculated by the Ministry of Labour up until 1947, but by then the 1914 fixed basket was becoming very unrepresentative of household expenditure.

The intention had been to update this cost-of-living index on the basis of a large-scale survey of household expenditure conducted in 1938-39, but the Second World War supervened, and it was not until 1947 that a new Interim Retail Prices Index was introduced, based on the 1938-39 survey. The Cost of Living Advisory committee further recommended that a regular series of budget collections be instituted as the basis for the construction of a permanent index.

By early 1955, sufficient information from the Household Budget Inquiry became available to allow the committee to formulate a new index. This became the first official Retail Prices Index (RPI) and began in January 1956. Among the changes brought in were an expansion of the scope of households included in the RPI from the working classes to all wage earners, but excluding very high and low-earning households. It also included the first serious attempt to measure owner-occupiers' housing costs.[3] The committee also recommended that the Household Budget Inquiry should become a continuous survey, which led to the creation of the regular Family Expenditure Survey (FES) from 1957. Once the survey was established the weights could be revised annually and this process, which still continues, began with a re-basing of the RPI in January 1962.[4,5]

As we have seen, the RPI was specifically constructed to measure the price changes experienced by wage and salary earners, with an annual re-weighting to reflect the introduction of new goods and changing patterns of expenditure. The basic philosophy and methodology established in 1962 have been maintained, with various changes, of which the more important have been:

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[2] Ministry of Labour and National Service, *Interim report of the cost of living advisory committee*, Cmd 7077, March 1947, Appendix 1.

[3] Office for National Statistics, *Consumer Price Indices Technical Manual, 2012*, Appendix 1: Historical Background to the Development of Consumer Price Indices in the UK.

[4] In doing so, the UK was at the forefront of international statistical practice. As late as 1987, only four out of 24 OECD countries were using an annually chained consumer price index. Many countries still rely on less frequent re-weighting. c,f, Peter Hill, Recent Developments in Index Number Theory and Practice, *OECD Economic Studies*, vol 10, 1988, p144.

[5] In 2001 the FES and the National Food Survey were replaced by the annual Expenditure and Food Survey, which was renamed the Living Costs and Food Survey in 2008 when it became a module of the Integrated Household Survey.

- Changes to the method of calculating owner-occupiers' housing costs, including the use of mortgage interest payments from 1975 and of depreciation costs from 1995,
- The introduction of "seasonal weights" for fresh fruit and vegetables in 1975 and their discontinuance in 2008,
- The inclusion of the costs of holidays abroad and in the UK from 1993/94,
- Changes to the detailed definitions and coverage in the RPI in 1987 and in 1996. (In 1996 there were also significant changes in the locations and outlets from which prices were collected, designed to be more representative of shopping habits).
- Changes in 2010 to the collection of clothing prices, particularly around sale periods, designed to correct a previous systematic under-estimation of clothing price inflation.

*Construction of the RPI[6]*

Like most consumer price indices, the RPI aims to establish the cost of a representative basket of consumer goods and services in the base period and then to compare that with the cost of the same basket of goods and services in subsequent periods. Ideally, the basket would contain as wide as possible a range of goods and services: in practice, about 700 items are included, either because they are important in themselves (e.g. retail electricity prices) or because they are taken as being representative of a wider class of goods (e.g. the price movements of 17 items of women's' clothing, such as "dress", "formal skirt", "casual skirt", are taken as being representative of the entire section of "women's' outerwear").

The price indices calculated for the representative items are aggregated, by taking a weighted arithmetic average on the basis of the previous year's Living Costs and Food Survey of household expenditure, to obtain the price indices for the 85 sections. For analytical purposes, these are then further aggregated into groups, before the final aggregation into the overall RPI. The weights (parts per thousand) of the RPI groups in 1987, 1996 and 2013 are given below:

| Table 1: RPI Groups | 1987 | 1996 | 2013 |
|---|---|---|---|
| Food | 167 | 143 | 116 |
| Catering | 46 | 48 | 47 |
| Alcohol | 76 | 78 | 61 |
| Tobacco | 38 | 35 | 30 |
| Housing | 157 | 190 | 254 |
| Fuel and light | 61 | 43 | 43 |
| Household goods | 73 | 72 | 60 |
| Household services | 44 | 48 | 62 |
| Clothing and footwear | 74 | 54 | 43 |
| Personal goods & services | 38 | 38 | 40 |
| Motoring expenditure | 127 | 124 | 122 |
| Fares and other travel costs | 22 | 17 | 18 |
| Leisure goods | 47 | 45 | 30 |
| Leisure services | 30 | 65 | 74 |

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[6] This section is based substantially on, Office of National Statistics, *Consumer Prices Index Technical Manual*, 2010 edition, supplemented by earlier and later editions and by on-line ONS databases.

The current base period, where the RPI is set equal to 100, is January 1987. Every month up to January 1988, prices for each of the 700 items were collected and the RPI for that month was calculated.

In January 1988, new weights for each of the items were calculated, and the subsequent monthly item price indices were calculated and then aggregated on the basis of the new weights.

To produce the 1987-based indices, the indices were *chained* together using the overlapping calculation for January 1988. In the UK it is not the practice to chain the item indices, but only the higher-level section indices. Thus for May 1988 we have

$$I_{May1988/Jan1987} = \frac{I_{Jan1988/Jan1987}}{100} \times I_{May1988/Jan1988}$$

And so on for subsequent years.

So far, the principle of construction of the RPI is the same as that for all the other UK consumer price indices, and, indeed, for almost all international indices. It is a Laspeyres-type, annually chained, base weighted index, with upper-level aggregation on the basis of taking a weighted arithmetic average of the lower-level indices.

However, there is still the important and more controversial question of how the individual item indices are calculated. This involves price sampling and lower-level aggregation.

*Lower-Level Aggregation in the RPI*

Establishing the average price of one of the items in the RPI is quite a detailed process. It is simplest in the case of an item that is relatively homogeneous, such as the cost of an MOT test for a family car. In a case like this, it might be thought that there is systematic variation between regions and the price collection process would be *stratified* by region, with separate average MOT prices calculated for the North-East, London, etc.[7] Each of these would be an *elementary aggregate*, calculated by choosing, say, twelve garages in the North-East and establishing the price they charge for the MOT by visiting them, or in some cases, by telephoning, on the monthly Index Day.[8] The average price for an MOT in the North-East in that month is then taken as a simple arithmetic average of these prices.[9] The same garages are visited each month in the year to ensure comparability (although the regional sample is refreshed every four years or so). The price index for that month, relative to January, is calculated as the ratio of the average price for that month, relative to the average price for January. This method of calculating the elementary aggregate price index is called the *Ratio*

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[7] For some items, there is, separately or additionally, stratification by type of outlet – e.g. supermarket or independent.

[8] In total, about 120,000 prices are collected each month, covering the 700 items, although some of these can be collected centrally.

[9] It might be thought that a more representative average could be obtained by weighting the price quoted by each garage by the number of MOTs it issues. However, attempting to collect data on the monthly quantities sold for each of the 120,000 goods or services priced would be so burdensome to the businesses concerned as to be impossible. With a few exceptions, therefore, elementary aggregates are always calculated as *unweighted* averages. Where stratification is by type of outlet, an element of weighting of price quotes can be achieved based on the major supermarkets' market share.

*of Averages*.  (In the international statistical literature this method is sometimes referred to as the *Dutot*[10]).  It is used for 46% of the items in the RPI, accounting for 30% of expenditure.

The Ratio of Averages is the most straightforward method to use, and is, in fact, the most widely used formula internationally in lower level consumer price aggregation,[11] and it works well where the individual items are homogeneous and tightly defined (e.g, a 1 lb loaf of sliced white bread).  However, for many items, there is a wide variety in the type and quality of goods that fall within the item definition.  For example, if our price samples for a dining table came from IKEA and Heal's, a simple, unweighted average price would be dominated by the more expensive table from Heal's:  if the Heal's table doubled in price, then the overall index, calculated as the Ratio of Averages, would also approximately double in price, regardless of what happened to the price of the IKEA table.  In a case like this, the RPI uses the *Average of Relatives* method (sometimes referred to as the *Carli*[12]).  This is obtained by first taking the price of each product in the current month relative to its price in the base month, and then taking an arithmetic average of these price relatives.  In our example, this means giving an equal weight to the relative price change in the IKEA and Heal's tables. Mathematically, the price index at time t, relative to time 0, calculated as the Average of Relative prices is expressed as:

$$I_{t,0} = \frac{1}{n} \sum_{i=1}^{n} \frac{p_{i,t}}{p_{i,0}}$$

Whereas the Ratio of Averages works well if the sample prices in the base period are approximately equal, the Average of Relatives method is most accurate when the expenditure on the sampled products is approximately equal in the base period.  (In our example, when the higher price of the Heal's table is offset by the smaller number sold, so that it is reasonable to give the price relatives equal weight).  Compared to other countries, the UK makes more extensive use of the Average of Relatives method.  This is because the UK has always been concerned to make sure that the sampled prices are representative of the item in question and has, therefore, adopted relatively wide item definitions.  For example, in the construction of the French consumer price index, the item "men's' formal shirt" would be very tightly defined as to material, colour and quality, so that an index calculated as a Ratio of Averages would give a very accurate result for the price movement of such a shirt, but one would be less confident that such an index represented the price movement of shirts in general.  In the UK, the enumerators would be encouraged to sample a shirt typically sold in the chosen outlet.  Thus, the UK price relatives would be based on the price movements of designer shirts sampled in a boutique, mid-price shirts bought in a department store, and cheap shirts bought at a discount outlet.  A price index calculated as the Average of Relatives would then give a good indication of the price movement of shirts in general, although with

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[10] being the method used in a book published in 1738 by the French political economist Charles Dutot .

[11] c.f. OECD, *Main Economic Indicators*, Consumer Price Indices Sources and Definitions, available at http://stats.oecd.org/mei/default.asp?lang=e&subject=8

[12] after the formula used by the Italian public economist Giovanni Rinaldo, count of Carli, in a treatise published in 1764.

greater sensitivity to the sampling procedures. In the UK, the Average of Relatives method is used for elementary aggregation for 39% of items, accounting for 27% of expenditure.[13]

## 3. The Consumer Prices Index

The Consumer Prices Index (CPI) is the name that was given in December 2003 in the UK to the Harmonised Index of Consumer Prices (HICP) when the Government adopted it as the target inflation rate to be used by the Bank of England. All European Union countries have been obliged to publish a national HICP since March 1997. Since then, some of them have modified their domestic consumer price indices to bring them closer in methodology to the HICP but, apart from Romania, all the other EU countries have continued to use their own national consumer price indices for both macroeconomic purposes and for measuring the cost of living.[14]

HICPs were developed by the European Union in order to assess whether EU member states fulfilled the inflation convergence criterion for joining the European Monetary Union, namely that HICP inflation should be no more than 1.5% higher than the unweighted arithmetic average of the similar HICP inflation rates in the three EU member states with the lowest HICP inflation.[15] Subsequently, there was a shift of focus from national HICPs and biennial Convergence Reports towards compiling the Monetary Union Index of Consumer Prices (MUICP), which is the centre of interest for the European Central Bank in its assessment of price stability in the euro zone.[16]

The purposes for which the HICP was developed have shaped its characteristics:

– It is an index designed to reflect monetary conditions – i.e. it aims to capture all monetary expenditure on consumer goods and services in a country,

– It aims to be as closely comparable as possible between countries: it therefore mandated the statistical methodology in most common use in the EU countries, while omitting owner-occupied housing expenditure (where the variation between countries was too great to allow agreement on a common measurement) and tolerating considerable variation in national practices in regard to re-weighting, chaining, quality changes and sampling procedures. The Commission has, however, continued to press for greater uniformity.[17]

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[13] Average prices and price indices for the remaining items, that use neither the Ratio of Averages nor the Average of Relatives for elementary aggregation are calculated directly as weighted averages of a few centrally-collected prices (e.g. public utility charges) or by another method (e.g. mortgage interest payments, housing depreciation).

[14] UK Statistics Authority, "Communicating Inflation," *Monitoring Brief 7/2010*, 6 December 2010, Appendix 6.

[15] The other convergence criteria relate to the budget deficit, the government debt to GDP ratio, the interest rate on long-term government bonds and exchange-rate stability.

[16] *Report from the Commission to the Council on Harmonization of Consumer Price Indices in the European Union* (COM(2000)742 final, 21.11.2000), p16.

[17] For example, the UK was obliged to change its method of seasonal adjustment in the CPI (and the RPI, since seasonal adjustment is applied to the same data used for both CPI and RPI) from January 2011 to comply with a Commission regulation. The ONS reported that the new method was internationally regarded as an improvement and that its implementation would, on average, lower the CPI by 0.07 percentage points and lower

– The political imperative towards European Monetary Union was an incentive to develop a method that minimised differences in measured inflation rates. There was also an incentive on individual countries seeking to join the European Monetary Union to adopt methodologies that lowered their HICP.[18]

*Construction of the CPI*

The characteristics of the CPI reflect its origin as an index designed to guide monetary policy for European Union countries. Thus:

In principle it covers all consumer monetary expenditure in the UK. It therefore includes expenditure by all households and institutional residents in the UK as well as expenditure by foreign visitors, and it excludes spending abroad by UK residents (e.g. on foreign holidays). Again in principle, it excludes non-monetary, imputed transactions, such as for benefits in kind, as well as transactions such as fees and property taxes that, in national accounts terms, are counted as transfers.[19]

It excludes almost all owner-occupier housing costs.

The upper-level construction of the index is on the same basis as the RPI, although with a different aggregation of items into classes, groups and divisions, based on the EU version of an international classification framework, "Classification of Individual Consumption by Purpose" (COICOP). The 1987 and 2013 weights, in parts per thousand, of the CPI Divisions are given below:

| Table 2: CPI Divisions | 1996 | 2013 |
|---|---|---|
| Food and non-alcoholic beverages | 156 | 106 |
| Alcoholic beverages and tobacco | 70 | 44 |
| Clothing and footwear | 67 | 68 |
| Housing, water, electricity and gas | 134 | 137 |
| Furnishings and household equipment | 90 | 59 |
| Health[20] | 7 | 25 |
| Transport | 154 | 148 |
| Communication | 21 | 31 |
| Recreation and Culture | 131 | 141 |
| Education | 11 | 21 |
| Restaurants and hotels | 111 | 117 |
| Miscellaneous goods and services | 48 | 103 |

An important difference between the CPI and RPI is that the CPI makes use of a different method of elementary aggregation for many of the items in the index. When the HICP was developed, Commission Regulation (EC) No 1749/96 effectively excluded the use of the

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the RPI by 0.04 percentage points. c.f. ONS, *Measurement of Seasonal Items in the Consumer Prices Index and Retail Prices Index*, CPAC(10)16, September 2010.

[18] Of the 15 EU countries in 1997, 9 adopted a different elementary aggregate in at least some of their HICP compared to usage in their domestic consumer price index – in all of the 7 for which an estimate was available, this lowered the HICP. c.f. Commission report COM(2000)742, op cit, Table 9.

[19] The CPI generally uses the same price sample as for the RPI, although there is different measurement and treatment of, most notably, insurance and new car prices.

[20] There is a break in the CPI series in 2001, when hospital, nursing and retirement home costs were included in the HICP for the first time.

Average of Relatives method for elementary aggregation because it could give different results from the other methods. It was, in any case, used by relatively few EU countries.[21] Instead, the two allowed methods were the Ratio of Arithmetic Means (referred to in the UK as the Ratio of Averages) and the Ratio of Geometric Means (referred to in the UK as the *Geometric Mean* and sometimes referred to internationally as the *Jevons*[22]). As we have seen, the Ratio of Averages method is excellent where the items are homogeneous. If they are not, then the Commission Regulation left the Geometric Mean as, in practice, the only feasible method of aggregation.

The most straightforward way to calculate an index using the Geometric Mean is to take the price of each product in the current month relative to its price in the base month, exactly as for the Average of Relatives, but then to take the *geometric* rather than the arithmetic average of these price relatives. (A geometric average of n numbers is taken by multiplying them together and taking the n$^{th}$ root). Mathematically, this can be expressed as:

$$I_{t,0} = \left( \prod_{i=1}^{n} \frac{p_{i,t}}{p_{i,0}} \right)^{\frac{1}{n}}$$

A possible rationale for using the rather unusual geometric mean rather than an arithmetic average will be discussed later in the section on formula bias. For the moment, the key point is that, as a matter of arithmetic, the geometric average is always less than the arithmetic average and consequently a price index calculated using the Geometric Mean will always show lower inflation than an index calculated using the Average of Relatives. Moreover, this difference is greater the greater is the variance in the price relatives. Because most EU countries had quite tightly defined items, the variance of their price relatives was relatively low, and for five of the seven countries for which estimates are available, a change to using the Geometric Mean for some or all elementary aggregation led to a fall in measured inflation of no more than 0.1 percentage points. In the case of Denmark, which, unusually, uses monthly chaining of its elementary aggregates, the estimated effect was between 0.3 and 0.4 percentage points, and in the UK, with its broad item definitions, the result was a fall in measured inflation of 0.65 percentage points. (This formula effect difference has since widened, in 2013, to an estimated 0.9 percentage points).

The CPI uses the Geometric Mean for 70% of items, accounting for 63% of expenditure. Aggregation using the Ratio of Averages is used for items accounting for 5% of expenditure, and the remaining items are aggregated using available product weights. It is not clear why the UK switched most of the items that use the Ratio of Averages in the RPI to using the Geometric Mean in the HICP, since it was not obliged to do so by the HICP regulations, nor were there any statistical grounds for doing so.

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[21] The average of relatives method was allowed for elementary aggregation if its use resulted in an index giving an annual inflation rate differing systematically by no more than 0.1 percentage points on average from the other methods.

[22] W S Jevons, later professor of political economy at University College, London, used the formula in a paper in 1862.

## 4.  Differences between the RPI and CPI

Given the differences in population coverage, the types of goods and services covered, and the method of calculation for the RPI and CPI, it would be expected that they would produce rather different results.  In fact the difference is so large and systematic as to call into question whether they can both be adequate measures of the movements in the purchasing power of money.

*Chart 1: RPI and CPI, annual average 12-month inflation rates, percentage points*
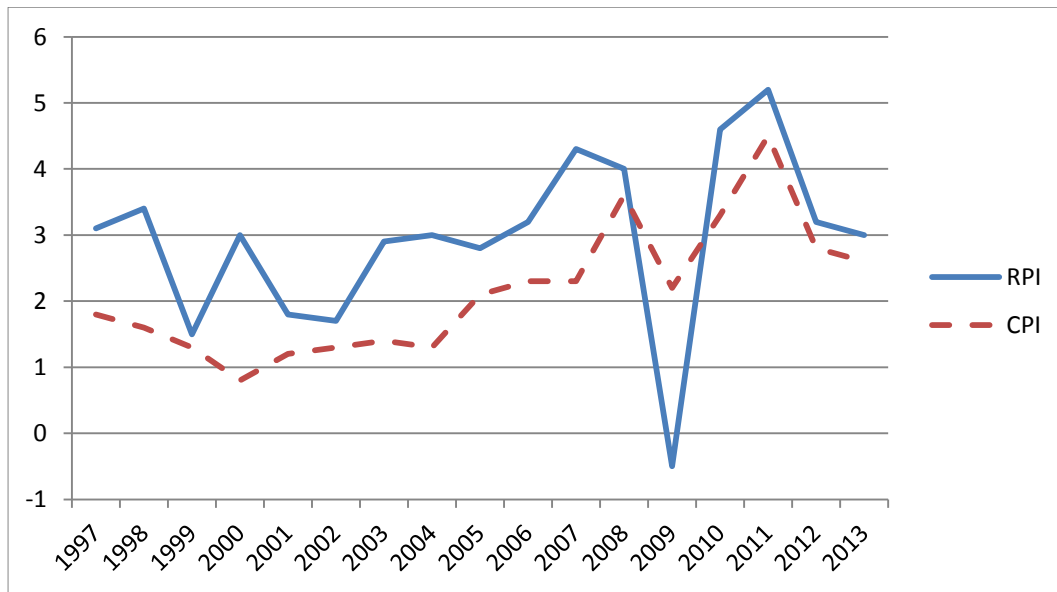


As Chart 1 shows (apart from 2009 when the cut in value added tax and the sharp reduction in mortgage interest rates caused the RPI to turn negative), RPI inflation has consistently been above CPI inflation, by an average of 0.8 percentage points between 1997 and 2013.

The difference is even more marked if one looks at the cumulative effect of the different inflation rates.  Chart 2 shows the annual average RPI and CPI indices, rebased to 1997 = 100.

*Chart 2:  RPI and CPI annual average indices, rebased 1997 = 100*
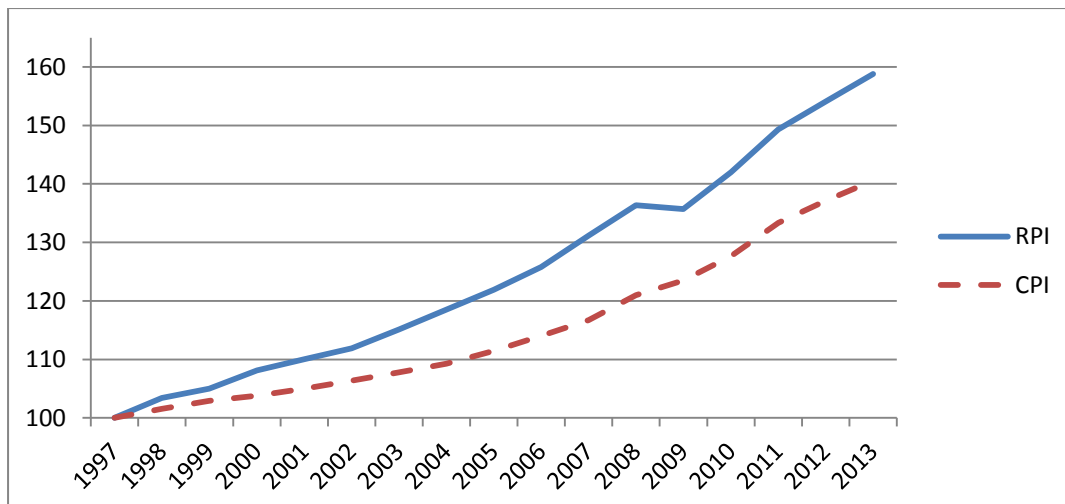
Over the sixteen years from 1997 to 2013, the RPI shows an increase in consumer prices of 58.8%, whereas the CPI shows an increase of 40.6%. This means that someone whose income had increased over the years in line with the CPI would need to have her 2013 income increased by 13% just to restore her purchasing power, as measured by the RPI, to its 1997 level. The three main causes of this difference are differences in coverage; the exclusion of owner-occupiers' housing costs from the CPI; and the effect of using different formulas for elementary aggregation. These are examined in turn.

## 5. Differences in Coverage

*Difference in Population Coverage*

As already mentioned, the RPI includes all private resident households in its calculation of expenditure weights (i.e. not those living in institutions such as student halls of residence, care homes and prisons), except for the top 4% of households by income, and pensioner households that derive more than three quarters of the income from the state pension. The effect of this is shown in the table below, with data from the Living Costs and Food Survey 2008-9.

*Table 3: Households excluded from the calculation of RPI weights*

*Top 4 per cent of households*
| | |
|---|---|
| Percent of all households | 4.0 |
| Percent of household expenditure | 10.5 |

*Pensioner Households mainly dependent on state benefits*
| | |
|---|---|
| Percent of all households | 5.3 |
| Percent of household expenditure | 2.2 |

Because the top 4% of households spend so much more than pensioners, the effect of these exclusions is mainly to place less weight on expenditure on goods typically bought by the richest households and, with more than 10% of household expenditure involved, the difference can be significant. There are, in fact, only a few items that are included in the CPI but entirely excluded from the RPI such as stockbrokers' fees, but the main effect is that the RPI gives a lower weight to luxury items such as private medical treatment and restaurant meals and a greater weight to consumables such as food, tobacco and take-away meals.

*Differences in Product Coverage*

The main difference in product coverage is that the CPI excludes almost all expenditure on owner-occupied housing. This is so important a difference that it is treated separately. But there are other, much smaller differences. The CPI, unlike the RPI, includes expenditure by foreign visitors and students in the UK and excludes spending by residents on foreign holidays. This may cause some volatility in the indices if, for example, exchange-rate movements cause the sterling cost of foreign holidays to change, or if the fees paid by foreign students are increased dramatically, but, generally such differences are much smaller than differences due to the differences in weights.

The Office for National Statistics produces a monthly analysis of the causes of the difference between the RPI and CPI. Its calculation of the effect of differences in coverage and weights between the RPI and CPI is produced partly as a residual, (once the effects of housing and the different aggregation formulae are allowed for). It is, therefore, subject to a greater margin of error than the other estimates.[23] Over the whole period since 1997 for which the CPI has been calculated, there has been no clear trend in the differences caused by these differences in coverage. Over the period as a whole, the average effect was to increase annual average CPI inflation by 0.21 percentage points relative to RPI inflation, but within this, there were periods up until 2004 and again in 2010-2011 when the effect was either very small or decreased CPI inflation relative to RPI inflation.[24]

## 6. Differences in the Treatment of Housing

The RPI measures owner-occupier housing costs by the "payment" method, i.e. by the payments that owner-occupiers have to make in order to live in their houses. Thus, it includes mortgage interest payments (but not mortgage payments that represent a repayment of capital); an estimate for depreciation, based on new house prices (to cover the periodic cost of maintaining the fabric of the building in "as-new" condition); building insurance and ground rent; house transaction costs (e.g. surveyor costs and estate agents' fees); payments for minor repairs and maintenance; council tax. Of these, the CPI includes minor repairs and maintenance but excludes all the other costs.

Owner-occupiers' housing costs that are included in the RPI but excluded from the CPI have a large weight in the RPI: 14.4% if mortgage interest payments are included, 11.5% if they are excluded. Since housing costs frequently move in a different pattern to other prices and historically have risen at a faster average rate, they are a large part of the explanation of the different pattern of short-term movements in RPI and CPI, and are the second-most important reason for the long-term tendency of the CPI to rise more slowly than the RPI. The ONS figures show that over the period January 1997 to July 2014, the exclusion of mortgage interest payments caused CPI inflation to be, on average, 0.07 percentage points lower than RPI inflation; and the exclusion of other owner-occupiers' costs caused the CPI inflation to be, on average, 0.42 percentage points lower than RPI inflation.

The exclusion of owner-occupiers' housing costs from the CPI (and the HICP, with which it is identical) is such an obvious weakness in what is supposed to a comprehensive consumer price index, that Eurostat, the statistical arm of the European Commission, has had a long-running but so far uncompleted project to develop an agreed measure for inclusion in the HICP. Meanwhile, in the UK the ONS has developed a measure of housing costs that differs

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[23] In 2005, the ONS changed its method of accounting for differences between the RPI and CPI, with the old and new methods (both of which were calculated from 2005 to 2010) occasionally producing differences as high as 0.2 percentage points in particular estimated effects for the same period.

[24] Coverage effects and the contribution of different product groups are discussed at length in Ruth Miller, *The Long-Run Difference between RPI and CPI inflation*, Office for Budget Responsibility, Working Paper No 2, November 2011. Her conclusion is that, in the long run, differences in coverage and weights (other than housing) will not cause a systematic divergence between RPI and CPI.

both from the RPI method and the method being trialled by Eurostat. Including this measure in the CPI gives a new, experimental index called CPIH. This is assessed later, in section 14.

## 7. The Formula Effect

The final and most important cause of the difference between the RPI and CPI is their use of different elementary aggregation formulas for many of the items in the index. Specifically, the fact that the CPI uses the Geometric Mean for items accounting for 63% of expenditure means that over the whole period since 1997, the CPI inflation has, on average been 0.62 percentage points lower than RPI inflation. However, this masks a change in the strength of the formula effect. In the period from 1997 to 2009 the formula effect was, on average, 0.51 percentage points. From January 2010, when new, wider, item definitions were introduced for clothing, until July 2014, the formula effect has averaged 0.91 percentage points.

Differences between the RPI and CPI caused by differences in population and product coverage are easily understandable and it is possible to argue that the different coverage is legitimate when the indices are used for different purposes – the RPI for uprating purposes generally and particularly for wages, and the CPI as a monetary policy indicator. However, that such a large difference should be caused by a difference in statistical technique is unsatisfactory, implying that at least one of the indices is flawed. We therefore look at the formula effect in more detail.

*Explaining the Formula Effect*

The first question to ask is why the Geometric Mean was introduced as a technique for elementary aggregation for the first time in the late 1990s, starting with Canada in 1995, so that, currently, about twenty out of the thirty-four OECD countries make at least partial use of the Geometric Mean for elementary aggregation.[25] After all, the Geometric Mean is rarely used in scientific work and almost never in everyday life: "the mean," unless otherwise qualified, always refers to the arithmetic mean.

There are, in fact, three approaches to the choice of elementary aggregation formula which have, at times, provided what seemed to be good reasons for switching to the Geometric Mean formula. These usually go by the names of the Economic Approach; the Stochastic and Sampling Approach; and the Axiomatic Approach. We shall deal first of all with the Economic approach, because it has been by far the most cited by statistical authorities as the reason for changing their elementary aggregation formula and also because, as we shall see, the Economic approach is also, to some extent, implicit in the other two approaches.

*The Economic Approach*

In the UK, the idea of consumer substitution inherent in the Economic approach was the first reason given by the Treasury in December 2003 for the switch of the inflation target from RPIX (the RPI excluding mortgage interest payments) to the newly-renamed CPI:

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[25] OECD Main Economic Indicators, *op cit*. The OECD reports are incomplete, with the number of countries, other than the UK reported as making some use of the Geometric Mean being eighteen, and with twelve countries reported as using only other formulae.

"the CPI better allows for the substitution of cheaper for more expensive goods and services within expenditure categories when relative prices change and so may be considered a more realistic depiction of consumer behaviour;"[26]

When in the June 2010 Budget the Chancellor announced that, from 2011, there would be switch to a system of uprating benefits, tax credits and public service pensions using the CPI rather than the RPI, he was quite open that the predominant motive behind the changes was "to put the whole welfare system on a more sustainable and affordable footing" and to improve government finances. But insofar as the government then or in subsequent Parliamentary debates put forward a statistical argument for the change, it was always on the basis that the CPI was better at allowing for consumer substitution.[27]

The relevance of the consumer substitution argument was summarised succinctly by the ONS in 2011:

"The actual level of consumer substitution in the market place is not clear. If consumer substitution behaviour is prevalent and considered to be appropriate in the inclusion of an index used for compensation purposes then the CPI provides the closer match to this ideal. If consumer substitution behaviour is limited or is not considered to be appropriate for inclusion in an index used for compensation purposes then the RPI provides a closer match to this ideal."[28]

As this quote indicates, it is a matter of opinion whether consumer substitution should be allowed for in compiling a consumer price index. Historically, most consumer price indices have been fixed-basket indices, where the price of a fixed basket of goods is calculated in the initial period and compared with the cost of the same basket of goods in a subsequent period. This is sometimes referred to as a cost-of-goods index. Of the three elementary aggregation formulae, only the Ratio of Averages or the Average of Relatives are appropriate for use in the calculation of such an index, since only they assume constant quantities of the products sampled (i.e. the maintenance of a fixed basket). By contrast, an index aggregated using the Geometric Mean is no longer a fixed-basket index, since the implicit weights vary during the year.[29] In the terminology of the UN's guide to producing consumer price indices, it is not a pure price index.[30] That is why in 2011 the ONS recommended the use of the RPI if consumer substitution is excluded.

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[26] Bank of England, *Remit for the Monetary Policy Committee of the Bank of England and the New Inflation Target*, December 2003, available at:
http://www.bankofengland.co.uk/publications/minutes/Documents/mpc/pdf/2004/remitletter2004.pdf
[27] c.f. speech by Lord Freud, Parliamentary Under-Secretary of State, Department for Work and Pensions, moving the Social Security Benefits Uprating Order 2011, HL Debates, 14 March 2011, vol 726, columns 75-78, 14 March 2011.
[28] Office for National Statistics, *Implications of the differences between the Consumer Prices Index and Retail Prices Index*, October 2011, p 4.
[29] This point had been made by the ONS when introducing the HICP. c.f. J O'Donoghue and C Wilkie, Harmonised Indices of Consumer Prices, *Economic Trends*, 532, 1998, Appendix: Construction of Elementary Aggregates.
[30] United Nations Economic Commission for Europe et al, *Practical Guide to Producing Consumer Price Indices*, United Nations, New York and Geneva, 2009, section 13.25.

An index that allows for consumer substitution is based on a different premise. Its aim is to calculate a price index such that if incomes were increased in line with the price index consumer welfare (as a whole, averaging across consumers) would be constant. It proceeds from the observation that a single, representative consumer, when faced with changing prices will adjust her purchases, moving along her fixed demand curve. According to Economic theory, this means that the consumer will normally buy more of products whose price has declined or increased more slowly, and less of products whose prices have increased faster. Thus, in the second period, her consumption basket will have changed from what it was initially. Consequently, the argument goes, we should not base our index only on the initial period basket, but we should give equal weight to the initial and final period baskets. There are a number of such symmetric consumer price indices, of which the best known is *Fisher's Ideal Index* (sometimes just the Fisher Index[31]). Calculating a symmetric index cannot be done in real time, since data on final period expenditures can be known only with a lag. It turns out, however, that under certain circumstances – where consumer substitution is high, such that price and quantity movements always exactly offset each other, leading to constant expenditure on each product for any price movement – the Geometric Mean is a good approximation for the Fisher Index. Where consumer substitution is low, the Geometric Mean will, however, underestimate the Fisher Index. By contrast, the Average of Relatives will overestimate the Fisher Index, although not by much if consumer substitution is low. That is why in 2011 the ONS recommended use of the CPI if consumer substitution was to be allowed for AND if consumer substitution behaviour is prevalent.

Since the Geometric Mean always produces a lower estimate of inflation than the Average of Relatives, uprating consumers' incomes in line with such an index will leave them unable to buy the initial basket of goods. But that is the point of the exercise. Consumers have already shown that they will shift their purchases to products whose price has risen more slowly: if we give them enough money to buy the initial basket in the second period they will, in fact prefer to buy a different basket with more of the lower-priced goods, giving them a higher level of utility – higher welfare. They will have been over-compensated. The aim of a symmetric price index (approximated by a Geometric Mean) is to produce a constant-utility index (sometimes called, in this context, a cost-of-living index).

Some countries are quite explicit about this. When in 1996 the influential Boskin Commission recommended that the USA switch from using a type of Average of Relatives for elementary aggregation to using the Geometric Mean, it gave as its motivation the desire to move towards what it called a cost-of-living index.[32] In the UK, however, the ONS continues to maintain that the CPI is a cost-of-goods index, without explaining the contradiction.

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[31] after the American economist Professor Irving Fisher.
[32] Boskin et al., Final Report of the Commission to Study the Consumer Price Index, US Senate, Committee on Finance, 1996, available at http://www.ssa.gov/history/reports/boskinrpt.html#cpi6, Executive Summary, paragraph 4.

An explanation has been put forward by Erwin Diewert, who was asked by the ONS to advise them on consumer price indices in 2012.[33] His argument is that if we want a fixed-basket index there is no *theoretical* reason to prioritise the initial period basket over the final period basket, and thus we are led back to a symmetric index like the Fisher index. And, provided that there is a negative relationship between the observed prices and quantities purchased, the Geometric Mean provides a good approximation to the Fisher index. Consumer substitution might provide the most plausible explanation for such a negative relationship, but one would not explicitly be invoking it as a reason for choosing to approximate a symmetric index.

This leads us on to a fundamental flaw in the way the Economic approach has usually been expounded, as if we are dealing with a single, representative consumer with constant tastes, who is exposed to changing prices set by her suppliers. The market prices and quantities that we observe – it is an elementary proposition in Economics – are not, in fact, those faced by a passive consumer but are set by the interaction of supply and demand. Thus, if we observe the market price of a product and the quantity bought in the initial period and a different price and quantity bought in the second period, we are almost certainly not observing a movement along a fixed demand curve. In fact, there is no way of knowing what combination of demand and supply movements was the cause of the changes. In Economics this is called the *identification problem*. The only way it can be solved is to estimate demand and supply equations for the product in question and for all the other products that could possibly be substitutes or complements. This is a difficult enough procedure for a single product and inconceivable for the 100,000 or more products for which price quotes are obtained when constructing consumer price indices, even if one introduces plausible restrictions on the types of substitutability and complementarity that are allowed for.

One can, however, make some generalisations. One is that, where supply changes predominate, one is more likely to observe a negative association between price and quantity; and where demand changes predominate, one is more likely to observe a positive association between price and quantity. An ONS paper by J Winton and others[34] includes a simulation study where they construct a two-good demand and supply model and then investigate the effect first of 500 random supply-side shocks and then of 500 random demand-side shocks. (The relevant section of this paper is included as Appendix 2 of this note). As expected, the supply-side shocks give a mainly negative observed relationship between price and quantity and the demand-side shocks a mainly positive relationship. The point is that, allowing for both supply and demand influences, there can be no presumption that either a positive or a negative observed relationship between prices and quantities will predominate.

We have already looked at the case where price changes are caused solely by changes in supply and where there will usually be a negative relationship between price and quantity. In this case the Average of Relatives will overestimate the Fisher Index to a small extent when

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[33] W Erwin Diewert, Answers to Questions Arising from the RPI Consultation, *UBC School of Economics Discussion Paper 13-04*, Feb 2013, Section 2.

[34] J Winton, R O'Neill, D Elliott, Section 2.5 in *Elementary Aggregate Indices and Lower Level Substitution Bias*, ONS, March 2012. A shortened version without the simulation study with its coloured charts was published in the *Statistical Journal of the International Association for Official Statistics*, vol 29, no 1, 2013, pp 11-20.

consumer substitution is low but by more when consumer substitution is high. The Geometric Mean will be a good estimator if consumer substitution is high (price and quantity changes exactly offsetting) – but if consumer substitution is very high it will overestimate the Fisher Index and if consumer substitution is moderate or low it will under-estimate it.

By contrast, if we turn to the equally likely case where demand changes predominate, one will usually observe a positive association between price and quantity. It turns out that in this case the Average of Relatives will underestimate the Fisher index, regardless of the strength of consumer substitution, whereas the Geometric Mean will underestimate the Fisher Index by even more.[35]

These results are borne out by the ONS simulations. When price and quantity changes were caused by random supply-side shocks, the Average of Relatives was closer to the Fisher Index than was the Geometric Mean whenever the elasticity of consumer price substitution was less than about 0.5 (a weak negative association) and the Geometric Mean was closer when consumer price substitution was higher (a stronger negative association); whereas when there were random demand-side shocks, the Average of Relatives was almost always closer to the Fisher Index.

These results provide a comprehensive Economic argument in favour of the Average of Relatives. If we want a fixed-basket Laspeyres index, we should use the Average of Relatives rather than the Geometric Mean. And if our target aggregate index is a Fisher index, such as is recommended as a way of accounting for consumer price substitution, the Average of Relatives is closer to it than is the Geometric Mean about three quarters of the time, assuming demand-side and supply-side shocks are equally likely. Moreover, the differences between the Average of Relatives and the Fisher Index are quite evenly distributed: when supply-shocks predominate, it is above it, when demand-side shocks predominate, it is below it. By contrast, the Geometric Mean is on average slightly below the Fisher Index when supply-side shocks predominate and considerably below it when demand-side shocks predominate, and thus it is bound to show a negative bias.[36,37]

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[35] An algebraic demonstration of these relationships is included in Appendix 1, taken from Mark Courtney, *CPI and RPI Differences: the Formula Effect and the Identification Problem*, August 2011, available at http://ssrn.com/abstract=1816262. A shortened and revised version was published as: M M Courtney, Consumer price indices and the identification problem, *Statistical Journal of the IAOS*, vol 29, no 1, 2013. Two numerical examples are also included.

[36] Winton et al, *op cit*, Box 1 and Box 2.

[37] Professor Diewert, in his advice to the ONS, shows an awareness of these results (and supporting empirical evidence) for products with a positive association between price and quantity when he devotes section 9 of his paper to what he calls "fashion goods" and writes, "Thus if any kind of matched model price index is used, the resulting index will show a tremendous downward movement throughout the year." Moreover, he includes as examples of fashion goods not just women's clothes but automobiles, electronic games and movies. He does not mention that this downward bias will be greater for the Geometric Mean than for the Average of Relatives, but of his four immediate recommendations for the RPI, the two relating to the formula effect are that in general the Average of Relatives should be replaced by the Geometric Mean (which he justifies solely on axiomatic grounds, of which more later); and that fashion goods should be dropped entirely from the RPI. That is at least a consistent approach, but a consumer price index that excludes all classes of goods where demand-side changes are important will not be representative, and one which – against his advice – includes them will be downward biased.

However, this Economic argument is based only on theoretical reasoning supported by simulations that may be plausible but are still artificial. Its propositions need to be tested by the empirical investigations of the sampling approach. Possibly the most that might be said is that it can provide useful insights into explaining *why* the sampling results occur.

*The Economic Approach in Eclipse*

It had always been known that the Economic approach was limited by being almost entirely theoretical. The Economic arguments for using either the Geometric Mean or the Average of Relatives are presented as if we had the relevant data on prices and quantities and could use the expenditure-weighted versions of these formulae. But the only reason we have to go through a two-stage procedure in calculating consumer price indices, calculating the item indices first by elementary aggregation and then performing upper-level aggregation, is that we do not usually have quantity information below item level. We therefore have to use the unweighted indices and we cannot know *a priori* that the unweighted Average of Relatives is a good estimator of the weighted Laspeyres index or that either of them is a good estimator of Fisher's Ideal Index. That has to be determined empirically – see the section on the Sampling Approach later in this Note.[38]

So long as it was assumed (on theoretical rather than empirical grounds[39]) that consumer substitution was prevalent, statisticians like Professor Erwin Diewert who were advocates of the use of the Geometric Mean, were content to argue that the Economic Approach provided support for its use, but had always maintained that it provided only very weak support.[40] Professor Diewert was one of the principal authors of the International Labour Organisation's Consumer Price Index Manual, and a similarly qualified statement appeared there in Section 20.86:

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[38] An interesting attempt to extend the Economic approach has been made by Peter Levell, *A winning formula? Elementary indices in the Retail Prices Index*, IFS Working Paper W12/22, November 2012. Applying the principle of maximum entropy, he concludes that where we have no information about the appropriate quantities bought or budget shares we should assume that they are equal. If we assume equal quantities bought in the base period we are led to the Ratio of Averages; if we assume equal budget shares we are led to the Average of Relatives. However, we can use the same principle by looking at quantities bought or budget share in both the initial and current period: we are, therefore, aiming at a symmetric index, like the Fisher index. If we assume constant quantities bought in the initial and current period we have the special case where the Average of Relatives is a good approximation of the Fisher index; if we assume budget shares are constant in the initial and current period we have the special case where the Geometric mean is a good approximation of the Fisher index (this assumption would be valid if all changes were along a static demand curve with a high rate of consumer substitution). Levell, therefore has provided an elegant restatement of the Economic approach, rather than avoiding the need for empirical confirmation.

[39] What evidence there was on consumer substitution came from studies of upper-level aggregation, where quantity data was more readily available. e.g. Steven D Braithwait, The Substitution Bias of the Laspeyres Price Index: An Analysis Using Estimated Cost-of-Living Indexes, *American Economic Review*, vol 70 No 1, March 1980, pp 64 – 77. This paper has – for econometric studies – an unusually fine level of aggregation, having fifty-three commodities, divided into ten sub-groups. Braithwait finds that the overall level of substitution bias (i.e. the extent to which using a Laspeyres index to aggregate the 53 commodity sub-indices together over-estimates inflation) to be only about 0.1 percentage points a year relative to a cost index based on his estimated Linear Expenditure System model, which estimated the degree of consumer substitution for each commodity. Other peer-reviewed studies tended to show even lower substitution bias.

[40] e.g. W Erwin Diewert, "*Axiomatic and Economic Approaches to Elementary Price Indexes*, NBER Working Papers 5104, 1995.

**"20.86** The results in the previous section gave some support for the use of the unweighted Jevons elementary index over the use of the unweighted Dutot, Carli and harmonic indices, provided that the proportional expenditures assumption is more likely than the proportional quantities assumption. This support is very weak, however, since an appropriate item price sampling scheme is required in order to justify the results."

However, in recent years an expansion of the Economic Approach to include both demand and supply movements has shifted the theoretical arguments towards use of the Average of Relatives, and it also began to be argued by proponents of the RPI in the UK that there were cases where consumer substitution was low and that, even according to the guidance given in the ILO Manual, "...there may be cases in which little or no substitution takes place within the elementary aggregate and the direct Carli might be preferred."[41]

In these circumstances, with the Economic Approach pointing the other way, Professor Diewert changed his mind and concluded that the Economic Approach was incapable of offering even very weak guidance towards the choice of elementary aggregation formula, so that, "In retrospect, it was probably a mistake to include this material on the economic approach to elementary indexes in the *Consumer Price Index Manual*."[42]

Professor Diewert was engaged by the ONS to advise them on consumer price statistics and by October 2012, the ONS had reversed the position they had taken in their October 2011 note[43] and decided that the economic approach should not be applied to the choice of elementary aggregation formula.[44]

## 8.  The Stochastic Approach[45]

We turn now from the Economic Approach to the stochastic approach, which, while still theoretical, is more closely based on how statisticians think about estimating price indices.

The stochastic approach assumes that we are trying to estimate the average rate of inflation of a population – i.e. of all the products that are included in an elementary aggregate. Specifically, given a population of price relatives $(p_1^i/p_0^i)$ showing price in period 1 relative to the price in period 0, we want to identify and then estimate their mathematical expectation $E\left(p_1^i/p_0^i\right)$, which is the mean of (one plus) the rate of inflation in this population of goods

To do this, we have a sample consisting of a number of observations of the price relatives. Now, the price-relatives can always (except in some extreme cases) be described by a decomposition into their mean and an additive mean-zero deviation. This means that we can estimate $E\left(p_1^i/p_0^i\right)$ in an unbiased way by taking their sample analogue, which is the mean of the sample of price relatives, i.e. the Average of Relatives. By contrast, the Geometric

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[41] ILO, *Consumer Price Index manual*, 2004, section 9.37.

[42] W Erwin Diewert, *Consumer Price Statistics in the UK*, October 2012, p87.

[43] Office for National Statistics, *Implications of the differences between the Consumer Prices Index and Retail Prices Index*, October 2011, p 4.

[44] Office for National Statistics, *National Statistician's consultation on options for improving the Retail Prices Index*, October 2012, paragraphs 40 – 42.

[45] This section draws heavily on Peter Levell, *A winning formula? Elementary indices in the Retail Prices Index*, IFS Working Paper W12/22, Nov 2012, Section 5.

Mean of the sample price relatives will give an underestimate of the expected value of the population price relatives.

It is sometimes argued that we should, instead, focus on estimating the expected value of the natural logarithm of price relatives $E[ln(p_1^i/p_0^i)]$ . We would then look at the distribution of the logarithm of price-relatives and describe them by their mean plus an additive mean-zero error. This means that $E[\ln(p_1^i/p_0^i)]$ can be estimated in an unbiased way its sample analogue, which is the mean of the logarithm of price relatives, which equals the logarithm of the Geometric Mean. This statistical model of the evolution of price relatives has been used to justify the use of the Geometric Mean.[46]

However, by Jensen's inequality, the convex transformation of a mean is less than or equal to the mean after convex transformation. Taking the exponential (or antilog) is a convex transformation. Therefore, although the logarithm of the Geometric Mean is an unbiased estimate of the mean or expected value of the logarithm of the inflation rate; if we take their antilogs we find that the Geometric Mean is below the expected value of the inflation rate.

The conclusion, therefore, is that the Geometric Mean is biased downwards as an estimator of the population inflation rate. And this is the case regardless of how price relatives evolve or whether we focus on price relatives or the logarithm of price relatives.[47,48]

It is sometimes argued that we should abandon the usual statistical assumption that the sample price relatives should be given an equal weight, because they are not truly random. This is the basis of the criticism of Jevons, Edgeworth and Bowley made by John Maynard Keynes in *A Treatise on Money*, 1930, (p 71-78), which is quoted at length in Chapter 16 of the ILO Manual. It is worth quoting from the ILO Manual's assessment:

> "The main point Keynes seemed to be making in the above quotation is that prices in the economy are not independently distributed from each other and from quantities. In current macroeconomic terminology, Keynes can be interpreted as saying that a macroeconomic shock will be distributed across all prices and quantities in the economy through the normal interaction between supply and demand; i.e., through the workings of the general equilibrium system. Thus Keynes seemed to be leaning towards the economic approach to index number theory (even before it was developed to any great extent)."

In his investigation of the value of money, Keynes was focusing on the determination of the purchasing power of money by looking at the price movements of commodities, i.e. at upper level not elementary aggregation, and it is now well accepted that where we have expenditure

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[46] e.g. W Erwin Diewert, *Consumer Price Statistics in the UK*, October 2012, pp 14,15. The ONS has conceded that its 2012 discussion paper that also used this argument was mistaken: ONS *Response to National Statistician's consultation on options for improving the Retail Prices Index*, February 2013, p 14. The ILO, *Consumer Price Index Manual*, 2004, ch 20 section 20.103 makes the same error, but ch 16, footnote 49 correctly argues from Jensen's inequality that the Geometric Mean provides a biased estimate of the expected value of (one plus) the inflation rate, although it mistakenly notes a positive rather than a negative bias.

[47] The ONS *Response*, op. cit., observes that the Geometric Mean is a consistent estimator of the *median* of a lognormal distribution, but it is hard to see any theoretical reason for adopting the population median as our measure of the average inflation rate. In any case, in practice, because many observed product prices are unchanged from one period to the next, the population median rate of inflation would very often be zero. See also the discussion of consistency in aggregation under the Axiomatic Approach in Section 10.

[48] Of course, because a sample estimator gives an unbiased estimate of a population target, it does not necessarily mean that it always provides a close estimate. This issue is discussed under the sampling approach.

data they should be used to weight the observations. But it is a different matter when we come to elementary aggregation. The only way to proceed, if we want a weighted index, is to make *assumptions* about how prices and quantities are related (which we may, indeed, occasionally be able to test).

One set of assumptions would take us back to the arguments of the economic approach. However, as we established earlier, in general we have no reason to think that either supply or demand changes would predominate in such a way as would justify moving away from the unweighted stochastic approach.

An alternative approach would attempt to get closer to a weighted index by purposive rather than random sampling, either assuming or attempting to give greater weight to observations that are thought to be more accurate estimators of the population inflation rate, e.g. sampling in proportion to expenditure (probability proportional to size sampling). That moves us on from the stochastic approach towards its practical application in the sampling approach.

## 9. The Sampling Approach

The sampling approach starts with the observation that the only reason why we need a two-stage estimation procedure is that we do not have data on the sale quantities associated with our price samples. If we did have such data we could construct our choice of overall price index based on the product expenditure weights in a single stage. As it is, however, we first of all have to perform elementary aggregation based on some unweighted aggregation formula, and we then calculate an overall price index by aggregating the weighted item price indices.

The first question therefore is, if we had data on the price and quantity sold of every product in the economy, what price index would we use for elementary aggregation? This then becomes our *target* index, and we investigate which of the unweighted elementary aggregation formulae provides the best estimate of it. Statisticians have developed a number of possible target indices, but in practice only two are in the running, the base-weighted Laspeyres index and some form of superlative or symmetrical index, which gives equal weight to expenditure in the base and current period. Since the alternative symmetrical indices give approximately similar results, we shall focus on the Fisher index.

A strong argument in favour of choosing the Laspeyres index as the target index for elementary aggregation is that consumer price indices at the upper level are almost universally calculated as Laspeyres indices.[49] Thus, if we could calculate our consumer price index in a single stage, we would do so as a Laspeyres index. Moreover, the Laspeyres index is *consistent in aggregation*. In other words, if all the quantity data were available, a two-stage procedure whereby both elementary aggregation and upper-level aggregation were

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[49] There have been economists who have argued that upper level aggregation should be by formulae other than the Laspeyres, but they have usually based their case on a variant of the now-discredited consumer-substitution argument. In any case the choice is constrained because only the base-period weights are available in real time, so the possibility of a usable symmetric index for upper-level aggregation does not normally exist.

performed according to the Laspeyres formula would give the same result as if the overall index were calculated directly in a single step.

By contrast, a Fisher index at the elementary level is not consistent in aggregation. The case for the Fisher as a target index at the elementary level has, however, been made on the basis that it is all right to use the Laspeyres at the upper level, because consumer substitution is low at that level and the annual re-weighting of the index allows for slow-moving changes in tastes, but that consumer substitution is high at the elementary level and therefore needs to be allowed for even at the price of inconsistency in aggregation. If this is done, the level at which elementary aggregation is separated from upper-level aggregation will affect the final result: in practice, this means that the number of elementary aggregates and the degree of precision with which they are defined will affect the overall consumer price index. But it has been argued that this inconsistency is not theoretically or empirically important.[50]

However, as we have seen, the Economic approach underlying the consumer substitution argument is flawed,[51] and one can no longer interpret a symmetric index as allowing for consumer substitution. It is hard to think of alternative reasons for choosing the symmetric, Fisher index as a target. One reason might be a preference for its axiomatic properties (discussed in the next section) over those of a Laspeyres Index,[52] since the Fisher Index satisfies the time-reversal axiom: however, except in special cases, it does not satisfy the transitivity axiom, which is what is important in practice.

Nevertheless, because researchers have taken the Fisher index as a target, we shall look at both the Laspeyres and Fisher indices as target indices at the elementary aggregation level and see how well the various elementary aggregation formulae perform. There are two criteria that statisticians use to determine whether a sample statistic (in this case the Average of Relatives or Geometric Mean) is a good estimator of a population target (in this case, the Laspeyres or Fisher index), namely whether it is efficient and whether it is unbiased. An efficient estimator is one which comes "closest" to the target – most commonly, statisticians assess this by its mean squared error (MSE). Except in certain risk-management applications, one also wants an estimator to be unbiased – i.e. to be as likely to overestimate the target as to underestimate it.

As an empirical test, the ONS was able to conduct a large-scale study on matched price and quantity data for sixteen sub-classes within the group of alcoholic beverages in the UK.[53] The population indices were first calculated using the available data for each of the

[50] ILO, *Consumer Price Index manual*, 2004, sections 9.46, 9.47.

[51] As a matter of fact, there is no *a priori* reason for thinking that supply shifts are relatively more important in explaining price variance at the sub-item level than at the upper level, since supply-side changes would impact fairly uniformly across an item, whereas changes in tastes might well affect products within the item differently.

[52] W Erwin Diewert, Answers to Questions Arising from the RPI Consultation, *UBC School of Economics Discussion Paper 13-04*, Feb 2013, Section 2.

[53] D Elliott, R O'Neill, J Ralph and R Sanderson, *Stochastic and Sampling Approaches to the Choice of Elementary Aggregate Formula*, ONS Discussion Paper, October 2012. The data comprised household-based purchase scanner data provided by TNS (now Kantor Worldpanel) and contained detailed descriptions of all commodities bought by a panel of approximately 25,000 members over 2003 to 2011, a total of 192,948 observations after outliers had been discarded. For the purpose of analysis it was aggregated monthly and also aggregated over regions and shop type.

elementary aggregates. The ONS then conducted a simulation study with samples drawn without replacement, based first of all on Simple Random Sampling and then on Probability Proportional to Size (PPS) sampling (with size determined by base period expenditure share from the population data). A sample estimate of the various population indices was then calculated according to each of the aggregation formulae. This sampling exercise was repeated 100 times and the mean squared error and bias estimated, for both sample designs.

With the Laspeyres index as the population target, the illustrative results for one elementary aggregate, European white wine, with a sample size of 10, are presented graphically in the chart below. "Time" on the horizontal axis refers to the month: because the bias and variance tended to increase over the nine years of data, the ONS estimated MSE and bias for months within a year using January of that year as a base month. The MSE and bias were then averaged over the years by month.

*Laspeyres as Target*
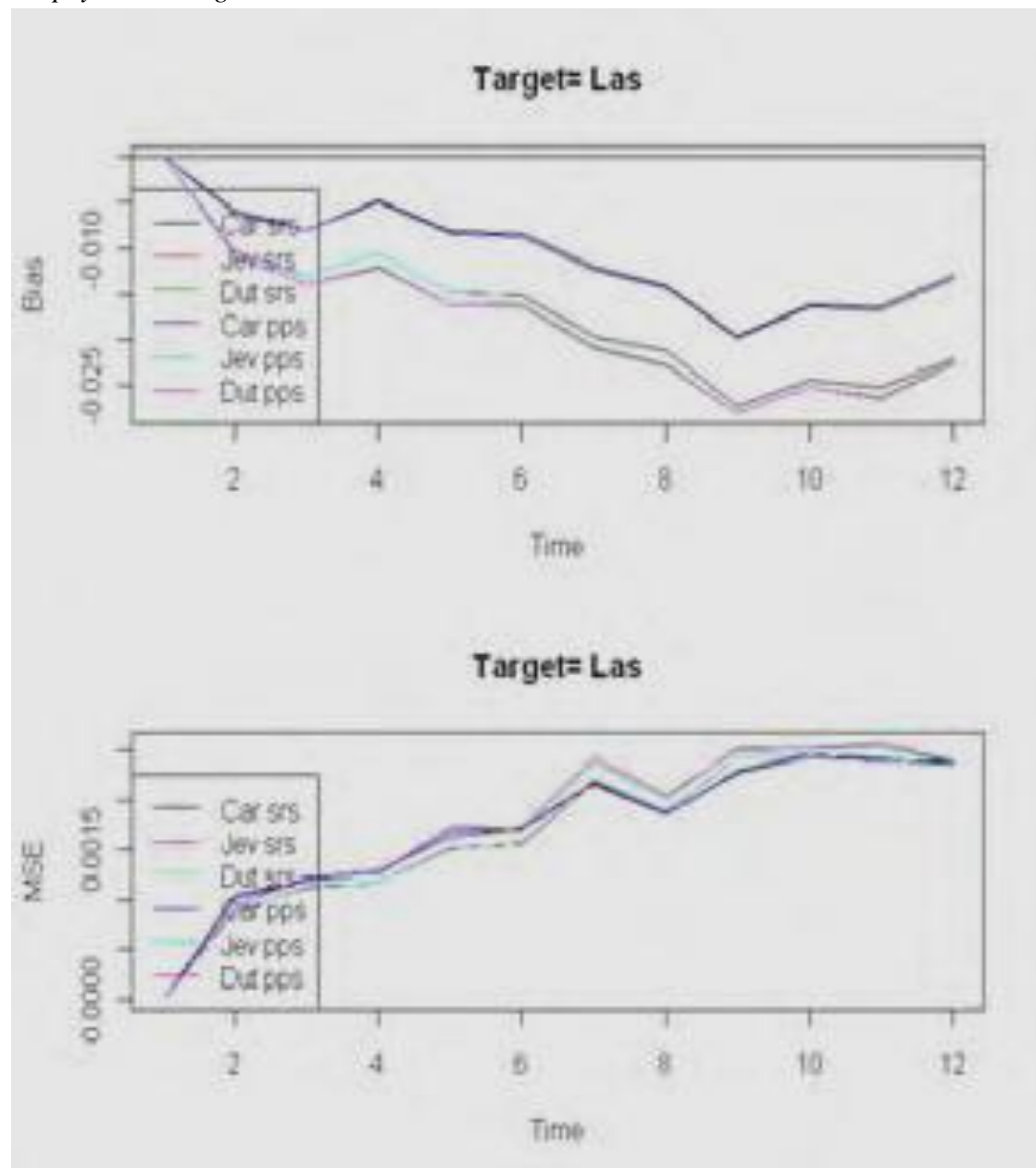
It turns out that the results based on simple random sampling and on PPS sampling were very similar (unsurprisingly, as European white wine is quite a homogeneous item), which makes the graphs easier to interpret. All the elementary aggregate formulae showed a negative bias, underestimating the Laspeyres index on average, but the downward bias of the Jevons (Geometric Mean) and Dutot (Ratio of Averages) was considerably greater than that of the Carli (Average of Relatives). And for this item, there was very little difference between the formulae in terms of the mean squared error.

When it came to presenting the overall results, the ONS concentrated on just the Average of Relatives (Carli) and Geometric Mean (Jevons) and compared their performance as estimators. The bold line in the chart below plots the average ratio of MSE(Jevons) to MSE(Carli) by sample size, where the sampling is by simple random selection. Here the average is both over time and over products (12 products had sufficient data to select samples of up to size 15). The Carli has a lower MSE for all sample sizes greater than 4, although, there is considerable variability by product and by time as is evident from the spread of the grey dots in the chart.
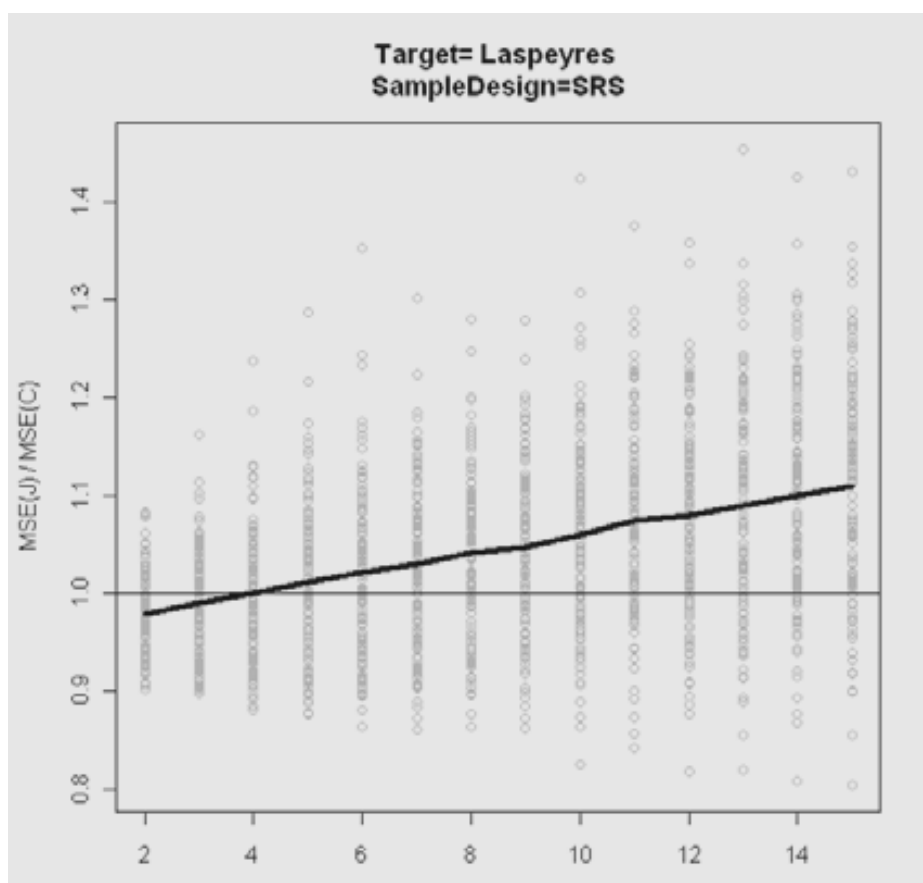


The next chart plots the coefficients of variation[54] and relative biases of the Carli and Jevons where the sample design is a simple random sample. As would be expected, the coefficient of variation decreases for both formulae, showing the benefit to accuracy of increasing the sample size. Also as expected, the relative bias does not change with sample size. The scale

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[54] The coefficient of variation is a measure of the dispersion of a frequency distribution. It is defined as the ratio of the standard deviation to the mean

of the chart makes it difficult to see, but the ONS paper reports that the average relative bias is -1.2% for the Carli (Average of Relatives) and it is -1.9% for the Jevons (Geometric Mean). In other words, both formulae on average underestimate the Laspeyres index, but the Jevons by more. For this data, the Average of Relatives has a smaller bias than the Geometric Mean when estimating the Laspeyres index (as would be expected from the stochastic approach) and, except for very small samples, it also has a lower mean squared error, and so, by any criterion it is a superior estimator.

Using the same data set but a different estimation technique, Mehrhoff[55] finds that, "the Carli index performs remarkably well at the elementary level of a Laspeyres price index, questioning the argument of its "upward bias" – in fact, it is the Jevons index that has a downward bias."



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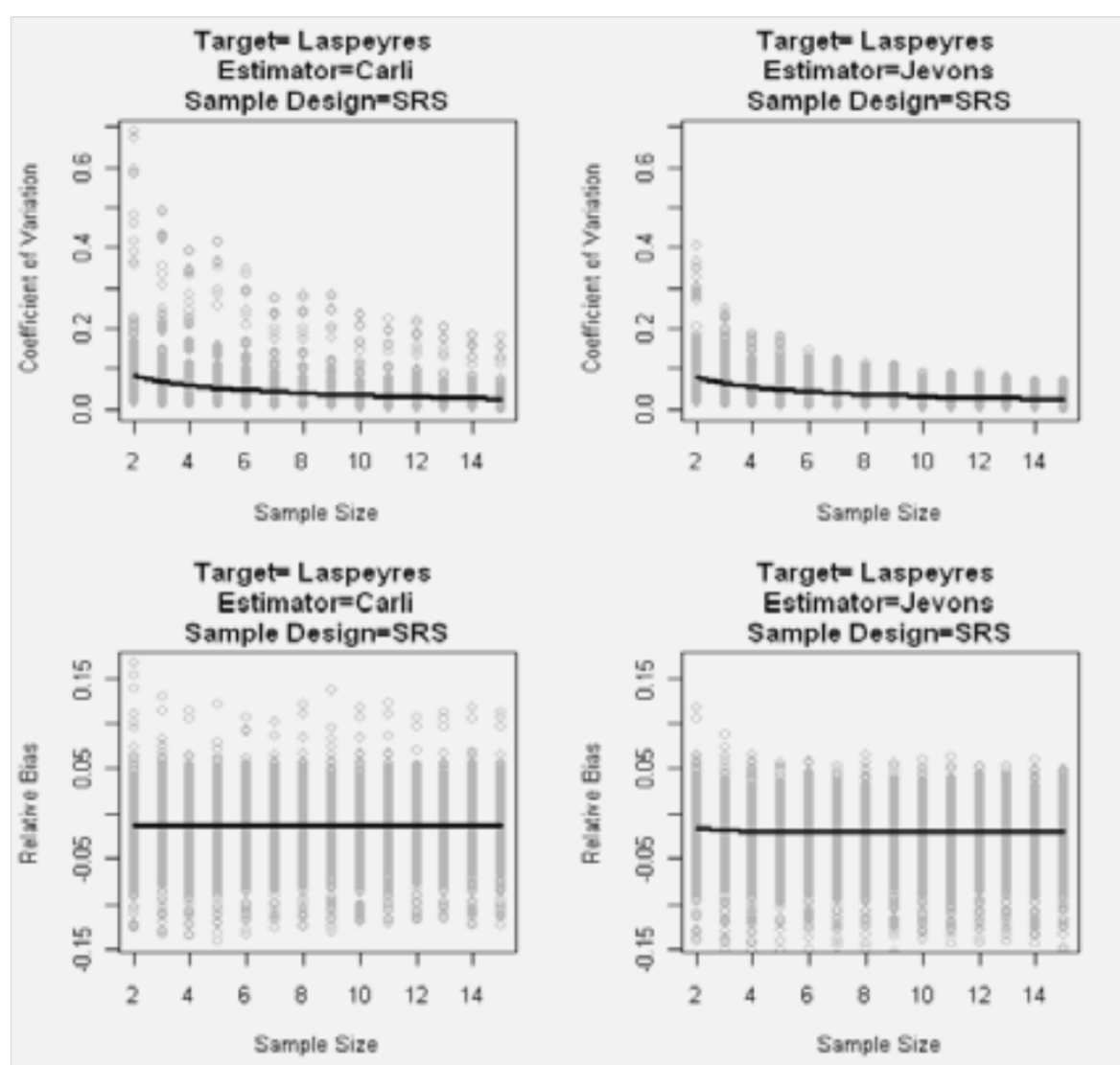
[55] Jens Mehrhoff, *The CIA (Consistency in aggregation) approach, a new economic approach to elementary indices*, Ottawa Group paper, 2 May 2013,
http://www.ottawagroup.org/Ottawa/ottawagroup.nsf/4a256353001af3ed4b2562bb00121564/8bdac0e73d96c89
1ca257bb00002fdb4/$FILE/Jens%20Mehrhoff%20efter%20konf%20The%20CIA%20approach.pdf .

*Fisher Index as Target*

We now look at the overall results when the Fisher index is the target. (The ONS noted that alternative symmetric indices such as the Tornquist and Walsh indices gave similar results).

The chart below shows that the Jevons had a slightly smaller MSE than the Carli, although the difference decreases with sample size. Again, there is considerable variability by product and by time.
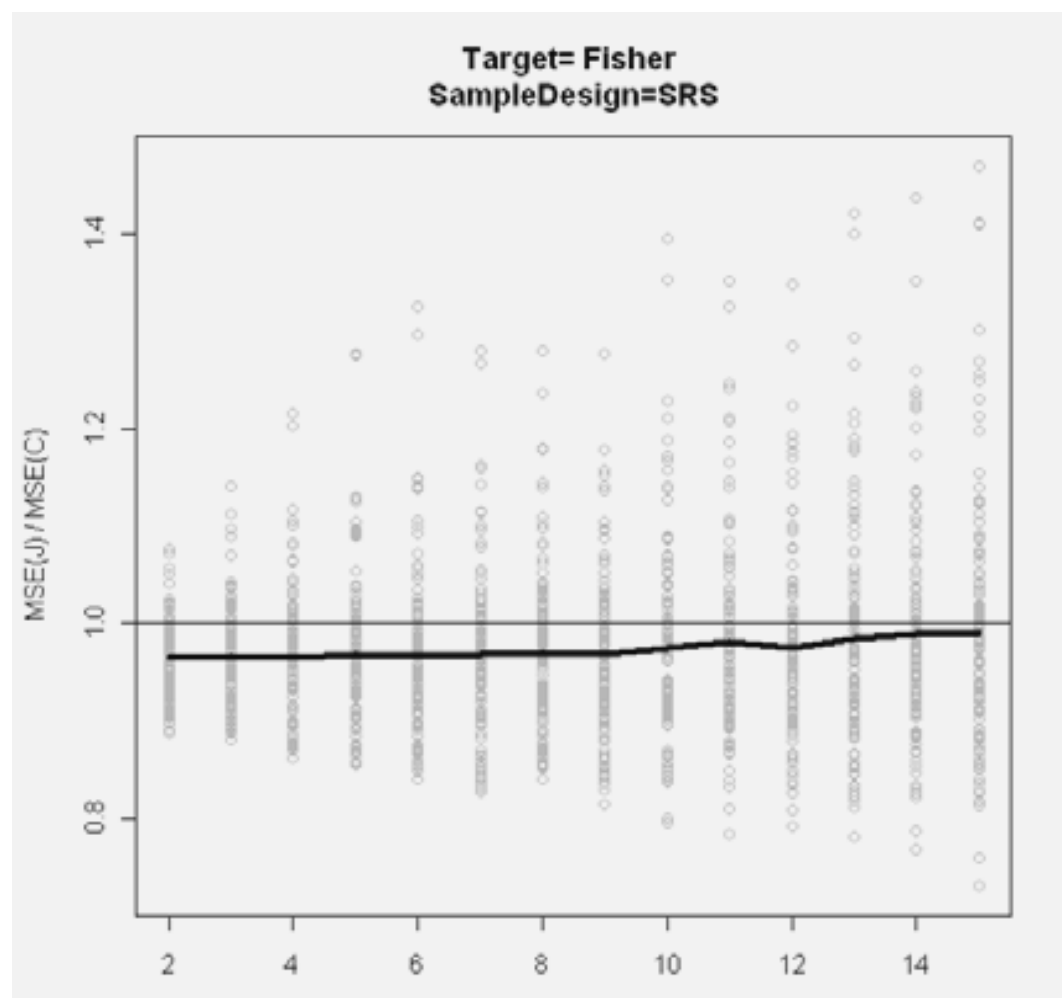


As shown in the next chart, increasing sample size decreases the coefficient of variance and increases the precision of the estimate. The Carli (Average of Relatives) shows a small upward bias of 0.33%. The Jevons a small downwards bias of 0.38%. Because of the narrowness of the differences between the two indices, both on MSE and on bias, there is little to choose between the two, even if one wished to assign a greater importance to either MSE or bias.
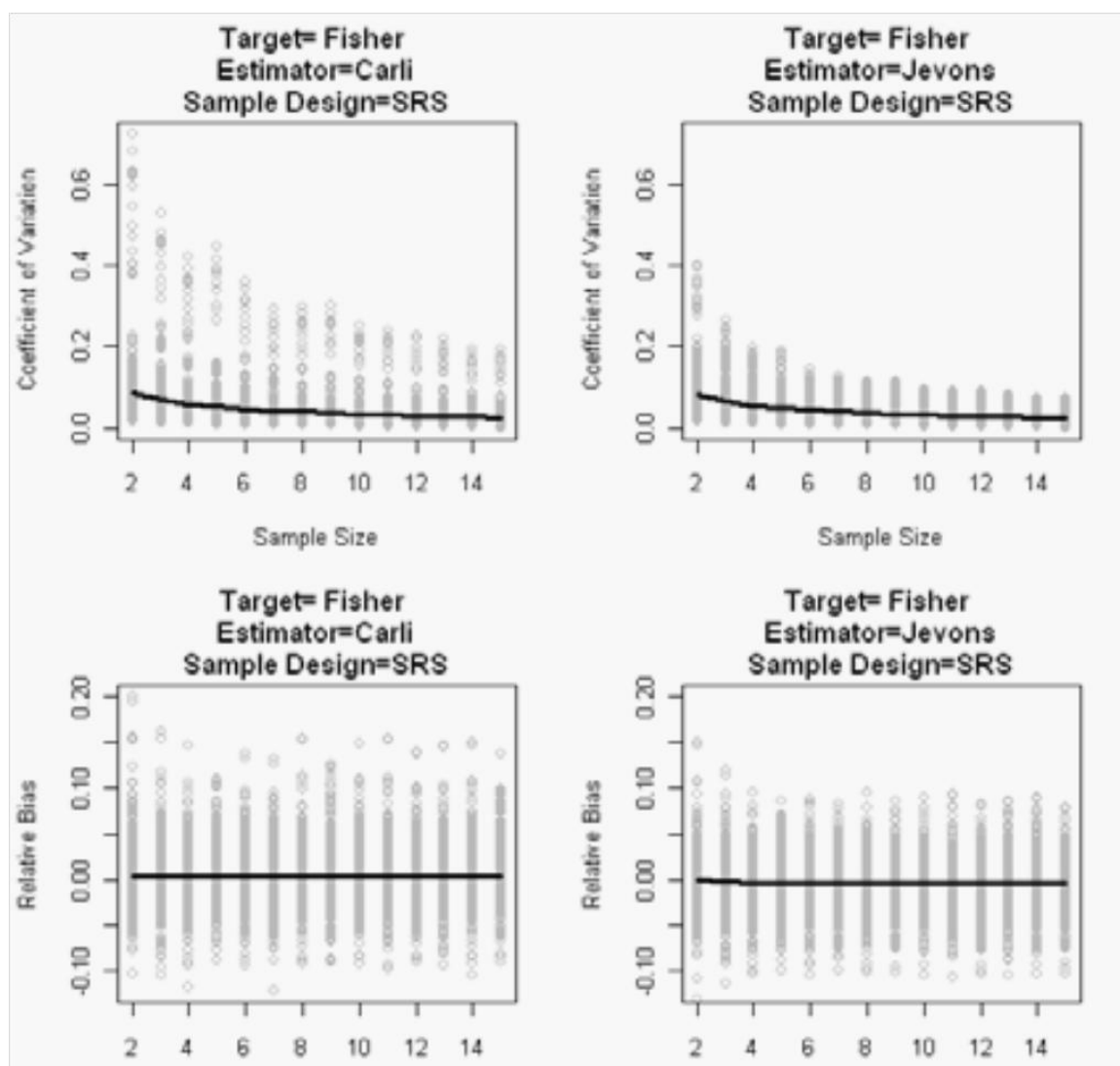
*Choice of target index when restricted to an unweighted index*

The ONS research team considered that the results for the Fisher index were data-dependent, "because there is no reason to believe that the average increase in prices as measured by an index that includes weighting information based on expenditure shares should show the same as a simple average such as the Jevons or Carli." In that case, one would need to choose an unweighted index as the target index. This makes their other research paper, in which they used the same alcohol data set, relevant.[56] Here, the ONS research team were not using a sampling approach, but were calculating various aggregation indices using the whole data set. For each aggregation index they therefore ended up with an index value for each period for each item. They then compared how closely they matched their target index, which they took to be the Fisher index. Using three alternative measures of closeness (different loss functions) they concluded that the Carli and Dutot were always closer to the Fisher than was the Jevons and that at the higher level of aggregation the Carli outperformed the Dutot. The implication is that if one has qualms about picking an expenditure-weighted index as the target index, but wants a target population index that best approximates the Fisher index,

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[56] J Winton, R O'Neill, D Elliott, *Elementary Aggregate Indices and Lower Level Substitution Bias*, ONS, March 2012.

then, at least for this data set, one would choose the Carli as one's population target index. (A more extended summary of the ONS research team's findings on this point is in Appendix 3).

The alcoholic beverages data had been chosen for investigation by the ONS because it was expected to show high levels of consumer price substitution. As already described, a wide range of substitution elasticities was calculated, but the average level, as expected, was quite high. (It should be repeated that, as the ONS researchers acknowledged, their assumption of static demand curves meant that they were, in fact, estimating the reduced form association between price and quantity, rather than consumer substitution). Thus, the alcoholic beverages data would be expected to provide a favourable basis for the contention that the Geometric Mean (Jevons) provided the best estimate for the Fisher Index, and its failure to do so was an important reason for the ONS to abandon the Economic Approach.

Another large data set with matched prices and quantities was acquired by the ONS for their clothing research. In this case, they were not looking at the effects of different aggregation formulae in constructing elementary aggregates but purely at whether substitution was taking place, and the effects of grouping products in particular ways.[57] The research found that estimates for the elasticity of substitution of the clothing items peaked at around zero, with a large proportion of estimates showing a positive correlation between price and quantity. The researcher's conclusion was that there was no evidence of substitution between brands. Again, particularly for clothing, we have to allow for the fact that what was being estimated was a reduced form relationship, rather than a demand function, so that the observed positive relationship between prices and quantities may be due as much to demand shifts as to brand loyalty. In either case – whether demand shifts predominate or whether demand is relatively static but with a substitution elasticity close to zero – the expectation would be that the Average of Relatives (Carli) would provide a much more accurate estimate of the weighted Fisher Index than would the Geometric Mean (Jevons), although this would have to be confirmed by calculations similar to those carried out for the alcoholic beverages data. Thus, although the Economic Approach cannot, in general, be used to choose which elementary aggregation formula to use, it can in some cases provide an indication. This issue is discussed in more detail in Mark Courtney, *The Economic Approach to Consumer Price Indices.*[58]

## 10.  The Axiomatic Approach

The fourth and last approach to choosing an elementary aggregation index is the axiomatic or test approach, which sets out a number of statistical properties that are desirable in an aggregation index – and the index that violates fewest of these axioms is deemed to be the best. Such an approach has little to recommend it for the following reasons:

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[57] Ria Sanderson, "Do Consumers Substitute Between Clothing Brands? Estimating the Elasticity of Substitution for a Selection of Clothing Products," *Survey Methodology Bulletin*, No 72, Spring 2014, pp 57 -72.
[58] Mark Courtney, *The Economic Approach to Consumer Price Indices*, March 2015, available at http://ssrn.com/abstract=2580423 .

i.     There is no agreement on what these axioms are – different authors have different lists and tend to downplay or omit altogether axioms that are awkward for their favoured aggregation formula,[59]

ii.     By Eichorn and Voller's non-existence theorem, there is no possible index that satisfies all the basic axioms,[60]

iii.     However, all the common aggregation formulae satisfy most of the axioms and

iv.     With one exception (commensurability), where they fail to satisfy an axiom, the effect has little empirical importance or can be minimised by sensible index construction.

One is left, therefore, with the impression that the advocates of this approach are, as Keynes said when criticising Fisher's test approach, more focused on algebraical elegance than on finding a formula that is a more probable approximation to the truth.[61]

However, among statisticians, if not the general public, the axiomatic approach has assumed an important role in the debate about elementary aggregation formulae and therefore needs to be looked at quite carefully.

Since most of the axioms are satisfied by all the common aggregation formulae, we shall concentrate on just four which are failed by one or other of the three elementary aggregation formulae.

*Commensurability*

Formally, this axiom states that the index should remain unchanged if we change the units of measurement of all the constituents of the index. This is not a problem for the indices that are based on price relatives, but it will be failed by the Ratio of Averages if the products in the index are not homogeneous. In our tables example, one would get one answer if the unit was just "dining table", but another answer if it was, say, "seating capacity of dining table", or "man-hours involved in producing dining table." In any case, we arrive at our previous conclusion that the Ratio of Averages does not work well for heterogeneous items.

*Positivity*

This simply states that the price index should always be positive. This seems a rather basic property but it is failed by the Geometric Mean. If we have a situation where the price of one of the sampled products drops to zero in the current period, then its price relative becomes zero, and the Geometric Mean, (formed by multiplying all the price relatives together and taking their $n^{th}$ root) also becomes zero. For example, if our item is "parking charges" and just one local council abolishes parking charges in one of its car parks, then the whole item becomes zero, as if all parking charges had been abolished.

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[59] c.f.Chapter 16.2, in the 2004 ILO *Consumer Price Indices Manual*: "different price statisticians may have different ideas about which tests are important, and alternative sets of axioms can lead to alternative "best" index number functional forms.... Hence the axiomatic approach can lead to more than one best index number formula."

[60] Peter Hill, Recent Developments in Index Number Theory and Practice, *OECD Economic Outlook* vol. 10, 1988, p 127.

[61] JM Keynes, *A Treatise on Money, vol.1: The Pure Theory of Money*, 1930, p 101.

Even if products only occasionally move from being charged for to being free, this might seem a worrying failing. But in fact all statistical agencies have some procedure for dealing with outliers, which removes price relatives that are atypical. Failing the positivity axiom is an extreme example of the property of the Geometric Mean that it gives a greater weight to price falls than to price rises, and this is relevant for how outlier adjustment ought to be performed.

*Consistency in Aggregation*

This has already been discussed under the sampling approach, where it was pointed out that the Fisher index, and for that matter the Geometric Mean, are not consistent in aggregation with the Laspeyres index that is almost universally used for upper-level aggregation.[62] However, it is unclear how important the lack of consistency in aggregation would be in practice, and one would have to look at the results from the sampling approach to see how good an estimator the Geometric Mean was for our target population index. One can envisage scenarios in which the Geometric Mean is, for example, a more efficient, although more biased, estimator of a population Laspeyres than is the Average of Relatives (which is consistent in aggregation).[63] Thus, the consistency in aggregation axiom should not necessarily rule out any elementary aggregation formula in advance.

*Transitivity*

The transitivity axiom, sometimes called the circularity axiom, states that the product of a chain of indices over successive periods should equal the price index calculated directly over the whole period. A special case of the transitivity axiom is the *time-reversal* axiom, where the end-period prices are identical to the initial-period prices (and thus it looks as if one has come back to the initial period). Any index that satisfies transitivity satisfies time-reversal. It is possible to satisfy the time-reversal axiom without satisfying transitivity, but time reversal is a theoretical curiosity, whereas transitivity is of practical importance for index construction, so we concentrate on that.

It turns out that the Average of Relatives fails the transitivity test, whereas the Geometric Mean passes it, and this has been used as a reason for preferring the latter formula.[64] More specifically, it is maintained that its failure of transitivity causes a chained Average of Relatives to be biased upwards and to over-estimate the true inflation rate. In its consultation about changing the elementary aggregation index in the RPI, the Office for National Statistics

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[62] The Geometric Mean is consistent in aggregation with the Geometric Laspeyres index, so it can be argued that it does not formally fail the Consistency in Aggregation axiom, even if the Geometric Laspeyres is never used in practice for upper-level aggregation.

[63] This is at the item level, where the same point can be made by saying that the Geometric Mean is sometimes less influenced by outliers (provided there are seldom price declines and provided outliers are not eliminated by validation procedures). However, if one is looking at a choice of elementary aggregation formula to be applied to a Group or to the whole consumer price index, then bias in the aggregation formula becomes the dominant consideration. Even if – which is unlikely – the Geometric Mean were a more efficient estimator than the Average of Relatives for a majority of items, a systematic negative bias would mean that it gave rise to a less efficient estimate for the Group or the overall index. c.f. the results reported in Appendix 3, where the Average of Relatives is on average more efficient than the Geometric Mean as an estimator of the Fisher Index at the item level but the difference in efficiency is even greater at the Group level.

[64] The Fisher index also fails transitivity, and as the Geometric Mean is usually justified as being a good approximation for the Fisher index, this reduces the force of the distinction.

put forward the failure of the Average of Relatives to satisfy transitivity as the sole statistical reason for shifting to the Geometric Mean.[65] And in its confidential memorandum to the Bank of England justifying the change of formula, the ONS went so far as to give the failure of transitivity as the only explanation for the whole formula effect difference between the RPI and CPI.[66]

In fact, failure to meet the transitivity axiom is no reason to reject the Average of Relatives, either on theoretical or on empirical grounds.

Theoretically, transitivity is part of the debate over whether and when to use a chained index rather than a direct index. At the upper level of aggregation it is attractive to choose a chained index, where information on quantities is used to update the weights and to introduce new commodities, but this will always result in some *chain drift*, where the resulting chained index differs from what a direct index, based on the original weights and commodities, would show. In other words, the evolution of a chained index is path dependent. Whether that path will typically be above or below a direct index depends on the way in which prices and quantities move together.

Bodhan Szulc, in an influential 1983 paper, addressed this question, looking at two polar cases. If prices and quantities tend to keep moving smoothly in the same general direction throughout the period covered then the chained Laspeyres will lie below the direct Laspeyres. The contrary case is that in which relative prices and quantities tend to oscillate. In other words, some commodities become relatively cheaper at first only to become relatively dearer again towards the end of the period, or vice versa. Szulc describes relative prices as 'bouncing". In this case, the chained Lapeyres index will lie above the direct Laspeyres. If some prices move smoothly while others bounce, the chain index may not differ significantly from its direct counterpart.

Peter Hill points out that, "Szulc's proofs rely on the standard assumption that movements in relative prices and relative quantities are negatively correlated, i.e. that we are observing the responses of price takers to movements in the relative prices with which they are confronted."[67] In other words, Szulc is using the now discredited consumer substitution assumption. If, we make the opposite assumption that prices and quantities are positively correlated (demand changes predominate over supply changes), then the influence both of smooth inflation and of price bouncing will be diminished. That is, the extent of chain drift will be diminished but its direction will not usually change. Szulc's general result still holds: the extent and direction of chain drift depends on the path taken by prices and quantities, with a presumption that smooth inflation will result in negative chain drift and oscillating inflation in positive chain drift. But there can be no *a priori* assumption that one is more likely than the other.

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[65] ONS, National Statistician's consultation on options for improving the Retail Prices Index, October 2012.

[66] ONS, *Addressing the Formula Effect – the Case for Change*, 31 August 2012, available at www.ons.gov.uk/.../freedom-of-information/.../rpi-consultation-freedom...or at https://www.whatdotheyknow.com/request/133451/response/352140/attach/4/Statistical%20arguments%20for%20change%20120830%20MFE%20paper%20v11.pdf

[67] Peter Hill, Recent Developments in Index Number Theory and Practice, *OECD Economic Outlook* vol. 10, 1988, p 136.

The argument so far has been in terms of the base-weighted Laspeyres index, but it still holds approximately in the case of the unweighted Average of Relatives, which is, indeed, consistent with it in aggregation.[68] It has also been pointed out, as was described when we introduced chaining in the RPI, that in the UK the item indices are not, in fact chained. Chaining takes place only at the upper levels. Since the item indices feed into these, they will still influence any chain drift, but there will be a degree of offsetting: inflation at the class level will be smoother than inflation at the item level, making negative chain drift considerably more likely than if the item indices themselves had been chained.

One final theoretical consideration concerns a hypothetical comparison, where prices do not show mean-reversion or bouncing within a shop, but where Shop A has first a low price and then a high price and Shop B has first a high price and then a low price. It is argued that this is a more likely or realistic scenario than straightforward time-reversal, and, confusingly, is sometimes treated as a separate "bouncing" axiom. The general opinion is that this is a very dubious procedure, since sampling is supposed to be on a matched basis by product and by outlet, and a product bought at one shop is not the same for consumer or supplier as the same product bought in another shop.[69]

The net effect of these theoretical considerations is that one returns to what was always the traditional view: that a chained index can drift above or below the direct index and that empirical evidence is needed.

There is, in fact, a remarkable dearth of evidence on this point. It is easy to make up examples to show positive chain drift in the presence of price oscillations – what Szulc in a later article called "artificial examples made up by the author for his 1983 paper to shock the reader." It is equally easy to make up examples where smooth inflation gives rise to negative chain drift. Numerical examples along these lines are presented in Appendix 4, showing both positive and negative chain drift. However, artificial examples like this do not take us very far.

Szulc himself published an empirical study in a 1995 article[70], in which he used monthly price data for over 50 commodities, collected from December 1988 to January 1994 in the province of Ontario, from which he was able to draw bilaterally matched samples. The number of monthly price figures used in the study ranged from 40 to 120 per commodity. Szulc reports results for only 24 of the 50 commodities: the basis on which the selection is made is not reported. He calculates chain indices for these commodities using various elementary aggregation formulae, where the chain link is made monthly, annually, or every five years (i.e. only once in the six-year sample). A direct index is not reported, so one

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[68] The results will hold exactly for the Average of Relatives on the assumption that expenditure on each product is constant throughout, i.e. that equal weights are always justified.

[69] Peter Levell, *A winning formula? Elementary indices in the Retail Prices Index*, IFS Working Paper W12/22, November 2012, Section 4 has an ingenious extension, whereby he tries to save a separate, new "bouncing" axiom by suggesting that shops first swap prices and then swap back, thus justifying a direct index showing zero price change. This is an unnecessary complication. If each of the shops is showing mean-reverting prices in this way, then there will be sufficient price-bouncing in Szulc's original sense to give rise to positive chain drift.

[70] Bohdan Schultz (Szulc), Choice of price index formulae at the micro-aggregation level: the Canadian empirical evidence, *Ottawa Group paper*, Oct-Nov 1995.

cannot read off the extent of chain drift: however, one can get an idea of what it would be by comparing monthly or annual chaining with the result of chaining only once in six years.

Generally, the results are as would be expected from Szulc's theoretical paper. Where commodities exhibit price oscillation, he finds some spectacularly large examples of positive chain drift using the Average of Relatives, but only when using monthly chaining and only, as he remarks, for commodities with price seasonality or subject to short-term special pricing strategy. These are precisely the sort of homogeneous commodities where short-term price bouncing would be expected to occur.[71] There is much less spectacular but still significant positive chain drift for annual chaining of such commodities. For these commodities, aggregating using the transitive Ratio of Averages or Geometric Mean shows little chain drift.

By contrast, the commodities with more stable pricing exhibit little annual chain drift when using the Average of Relatives, either positive or – for three of the commodities – marginally negative. The transitive Ratio of Averages or Geometric Mean formulae usually, but not always, show marginally less chain drift. For the transitive formulae, chain drift would be zero except for the effect of monthly or annual re-weighting and replacement of commodities. Similarly, in the case of the Average of Relatives, not all of the observed chain drift can be attributed to the formula used.

Szulc's emphasis on monthly chaining might seem odd but is understandable in the Canadian context, where, unusually, the consumer price index is calculated using monthly linking. In such a case greater importance needs to be attached to transitivity. Prior to 1995, the Canadians used the Ratio of Averages for elementary aggregation, switching in 1995 to the Geometric Mean.

When considering the implications of Szulc's results for the UK, it needs to be remembered that, first of all, the commodities that produced the largest positive chain drift in his study are all relatively homogeneous and are aggregated in the United Kingdom RPI using the Ratio of Averages, not the Average of Relatives. Secondly, the UK chains its indices only at the upper, class level: had Szulc calculated his annually chained results at the class level, his examples would undoubtedly have had smoother inflation and would have moved towards more moderately positive or even negative chain drift.

Studies that use a small selection of commodities are interesting, but ideally we would want to study the effect of chain drift on the consumer price index as a whole. To do so, we have to look at indirect evidence.

Gareth Jones used the published ONS figures for the formula effect difference between the CPI and RPI to estimate what part of the formula effect was due to price bouncing in the

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[71] A smaller study of Australian fruit and vegetable prices yielded a similar result, using monthly chaining over one year where, "the particular items chosen for this comparison typically exhibit price bouncing." K Woolford, *A Pragmatic Approach to the Selection of Appropriate Index Formulae*, paper presented to the Ottawa Group, 1994.

RPI.[72] His reasoning is that because price bounce between chain links cannot occur, and because January has been used for RPI chain-linking, the formula effects for January should on average be smaller than for other months of the year. His results show that this is so, but only to a very minor extent: the average formula effect for January, over a fifteen year period, was lower than for any of the other months, by only by an average of 0.02 percentage points. Jones notes that this estimated annual price bounce is within the rounding error of the published RPI and CPI.[73]

Jones adds that there may be some additional price bounce in both RPI and CPI generated above the elementary aggregate level, since both use the weighted Average of Relatives formula at these levels, but that most people have tended to regard this as small. Any price bounce here would be in addition to the formula effect and not part of it.

A further indication of the extent of chain drift in actual consumer price indices comes from the international experience with changing elementary aggregation formulae, which will be discussed more fully in a later section. In summary, with the exception of Canada with its monthly chaining and Denmark, which also uses monthly chaining of elementary aggregates, none of the countries gave price bouncing or transitivity as a reason for changing their elementary aggregation formula.

These theoretical results, and their rather meagre confirmation by empirical evidence, are generally well understood by statistical agencies.

In all the editions of the Consumer Price Indices Technical Manual up to the 2012 edition, the ONS wrote (emphasis added):

> "In the case of AR, it can be shown that **in certain circumstances** its use, when combined with chain-linking of the within-year indices, introduces a **small** upward bias in the overall price index." (section 2.4 in 2012 edition)

And

> "It is possible to chain an index **every month** rather than just every January. For RA, provided that the weights and item list remained fixed this would yield the same results. However, for AR the result would usually be that the index would grow more rapidly than it should, a phenomenon known as 'price bounce'." (section 9.4 in 2012 edition)

When the HICP was established in the European Union, Commission Regulation (EC) No 1749/96 introduced a comparability requirement and forbade the use of the Average of Relatives for elementary aggregation unless it produced an annual inflation rate differing systematically, on average, by no more than 0.1 percentage points from that produced using

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[72] Gareth Jones, paper deposited in Royal Statistical Society User Net Library, 12 April 2013, http://www.statsusernet.org.uk/communities/resources/viewdocument/?DocumentKey=1c753c1b-0af1-4a34-90eb-735404e4b35d

[73] There is a complication in that the CPI weights are updated in two stages, in January to take account of the new COICOP weights for CPI classes and above and then in February to take account of changes to the basket of representative items. This does not affect the principle behind Gareth Jones' test, that price-bouncing shows up only in the RPI and that only January will not show a price-bouncing effect, but, given the way in which the ONS calculates the formula effect difference between the RPI and CPI, the amount by which the average February formula effect is lower than the annual average should probably also be added to the price-bouncing estimate: this would increase it to 0.03 percentage points.

the Ratio of Averages or Geometric Mean. The HICP indices were allowed to be either direct or chained indices. The Commission forbade the use of chaining using the Average of Relatives if chaining was more frequent than annual, but allowed it for annual chaining. Annual chaining was, therefore, not seen as an additional source of divergence.

When in October 2012 the ONS consulted on changing the elementary aggregation formula in the RPI, it put forward a lack of transitivity in the Average of Relatives formula as the sole statistical reason for changing the elementary aggregation formula. However, when respondents challenged the ONS to demonstrate that this was empirically important by calculating both direct and chained versions of the RPI (which uses the Average of Relatives) and the CPI (which does not), the ONS declined to do so. In its response to the consultation the ONS said that it could not provide a numerical estimate of price bouncing "as the impact is included in the more general effect known as chain drift." In other words, the ONS conceded that any effect from price bouncing is so small as to be lost in the chain drift inherent in both RPI and CPI, caused by annual changes in the weights and the introduction of new products.[74]

Subsequently, in December 2014, the ONS published a paper giving some estimates for chain-drift using locally-collected CPI data (making up 57% of the weight of the CPI) for May, June and July in the years 2009 and 2010, introducing a break in June of each year.[75] Thus, for 2009 they were calculating price indices for July 2009 relative to the base month May 2009, both directly and as a chained index using June 2009 as the linking month. And similarly for 2010. They did this with elementary aggregation performed using the Jevons formula (as is done with all locally-collected CPI data); using the Carli formula; and using the Dutot formula. They then aggregated the elementary indices into classes using a Lowe formula similar (but not identical) to the Lowe formula used for upper-level aggregation in both the CPI and RPI.

The ONS calculated the chain drift (the difference between the chained and the direct index) for every class for each of the elementary aggregation formulae that were used. Their conclusion was that, "the use of a Carli formula at the elementary aggregate level does lead to greater chain drift than using a Jevons or Dutot formula – for most classes." This is unsurprising: the Carli is expected to provide greater absolute chain drift than the transitive Dutot and Jevons formulae and for <u>monthly</u> chaining, as used here, is more likely to be positive than negative.

The ONS paper says nothing about the absolute size of chain drift relative to the formula effect differences, and it specifically includes the disclaimer that "any numbers given here are not directly equal to any chain drift within the CPI." Nevertheless, the paper was released by the ONS to the RPI/CPI User Group on 22 December 2014, just before the publication of the Johnson review of UK Consumer Price Statistics on 8 January 2015 (i.e. without allowing

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[74] *Response to the National Statistician's consultation on options for improving the Retail Prices Index*, February 2013. The ONS response, by referring to "price bouncing" rather than "lack of transitivity", assumes that any effect, if measurable, would cause an upward bias.

[75] G Clews, A Dobson-McKittrick and J Winton, *Comparing Class level Chain Drift for Different Elementary Aggregate Formulae Using Locally Collected CPI Data*, ONS, December 2014.

time for its results to be reviewed) and was cited in the Johnson review[76] in support of the contention that the non-transitivity of the Carli index used in the RPI was a serious shortcoming. It is worth considering, therefore, whether the ONS paper can, in fact, tell us anything about the size of chain drift in the RPI.

The ONS paper presented its results in the form of diagrams or as averages across classes, so that it was not possible to see the results for particular classes. The data was subsequently made available to Gareth Jones and is reproduced in Appendix 5. As a preliminary, we should note that the data is given for CPI classes, all of which (for these locally-collected prices) use the Jevons formula for elementary aggregation. It is, therefore, not always possible to infer a single formula that would have been used for items in each of these classes in the RPI (which groups items differently): in some cases, the RPI would use a mix of formulae for items in one of the CPI classes. Nevertheless, the general results are clear.

It should be noticed that Gareth Jones extrapolates from the ONS estimate of the chain drift from <u>monthly</u> chaining to chain drift in the RPI and CPI using <u>annual</u> chaining, which would certainly be less. He is, therefore, giving a greater weight to the ONS estimates of chain drift than they actually deserve. Despite this, he finds that they do not provide any indication of serious chain drift in the RPI. He observes:[77]

"a) Most of the Carli chain drift is in the Food, Beverages and Alcohol classes, where the RPI uses the Dutot formula, so these figures are irrelevant to the RPI/CPI debate, since the Dutot formula does not share this chain drift.
b) There is one class, 10106 (Fruit), where all formulae display high chain drift. This is presumably a seasonal issue associated with choice of the 3 months May, June and July for the study.
c) The Clothing and footwear classes display moderate chain drift, but have a low weight (about 5%) and will therefore only contribute about 0.02% to the total formula effect of 0.9%.
d) There is some anomaly with class 90201 (Major durables for indoor and outdoor recreation). This affects all formulae in 2009, but does not occur in 2010.
e) Class 50101 (Furniture and Furnishings) shows high chain drift in both years. This class is worthy of more detailed research, including investigation of data collection practices.
f) Chain drift in the audio-visual sector (classes 90101-90105) is fairly low, but there appear to be some anomalies in the 2010 data.
There are however some indications of data quality problems in these results and these are worthy of detailed research by ONS."

Overall, Gareth Jones concludes that, "There is really no serious problem with chain drift in the RPI and the ONS paper is highly misleading. I am sticking to my original estimate that the difference in chain drift between RPI and CPI at the All Items level is about 0.02% per annum."

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[76] Paul Johnson, *UK Consumer Price Statistics: A Review*, UK Statistics Authority, January 2015.
[77] Gareth Jones, *Chain Drift Synopsis*, paper deposited in the Royal Statistical Society User Net Library, 6 February 2015,
http://www.statsusernet.org.uk/communities/alldiscussions1/viewthread/?GroupId=85&MID=2966#bm14

## 11. Summary and Overall Assessment of RPI and CPI as Uprating Indices

*Coverage*

As we have seen, in its coverage of both population and goods and services, the RPI was designed to provide a price index covering all the goods and services, including housing, purchased by British households, excluding the very rich and pensioners dependent mainly on state benefits. By contrast, the CPI, by excluding owner-occupied housing and council tax, as well as by aiming to cover all monetary expenditure only within the UK, produces an index that might be quite well suited as a monetary policy indicator, but which provides a defective measure of British households' purchasing power. Moreover, because of its different coverage, the CPI systematically under-estimated the inflation faced by British households by an average of 0.3 percentage points per annum over the last seventeen years since the CPI was introduced.

*Elementary Aggregation Formulae*

The RPI and CPI use different aggregation formulae for many of the items covered, the RPI using the Average of Relatives and the CPI using the Geometric Mean. As a matter of arithmetic, this means that, other things being equal, inflation as measured by the RPI will always exceed inflation as measured by the CPI. Because the UK has unusually broad definitions of its items, with consequent variations of inflation within items, this formula effect difference is unusually large in the UK: it has averaged 0.6 percentage points a year since the introduction of the CPI, but in recent years, following changes to the treatment of clothing, the formula effect has averaged 0.9 percentage points per annum.

It is sometimes argued that this formula effect divergence is not due solely to the differences in aggregation formula, but that part of it is due to the fact that both indices are "chained" each year to account for changes in tastes and the introduction of new goods, and that the RPI, because of its use of the Average of Relatives, will be subject to "price bouncing" over the chain link and will therefore an upward bias. There is, in fact, no theoretical reason why such "chain drift" should be positive rather than negative, and the available empirical evidence is that any such effect is very small − much less than 0.1 percentage points per annum.

Thus, the question is whether the average annual 0.9 percentage point formula effect difference is due to overestimation in the RPI, underestimation in the CPI, or something in between.

When the CPI was introduced, it was argued that the main explanation of the difference was in terms of consumer substitution. Uprating in line with the RPI would enable the consumer to buy the same basket of goods as a year ago. Uprating instead in line with the CPI would not be sufficient to do this, but it would enable the consumer to be as well off − to have the same utility − as before, because it was assumed that we were observing consumers substituting away from products that had gone up more in price towards products that had fallen in price or gone up by less.

In fact, the prices we observe are not the result of consumers moving along a stable demand curve in reaction to changes in supply. Price changes are the result of shifts in both demand and supply. And in that case, Economic theory indicates, even if we are aiming at an index that maintains constant consumer utility, using the Geometric Mean for elementary aggregation will usually result in an underestimation of inflation, whereas using the Average of Relatives could result in either over- or under-estimation depending on the particular changes.

However, Economic theory is an imperfect guide, since in practice elementary aggregation can only use a sample of prices, without knowing their importance within an individual item whose average price is being estimated. The stochastic approach used by statisticians shows that the Average of Relatives is an unbiased estimator of the average rate of inflation, whereas the Geometric Mean is biased downwards, regardless of the distribution or evolution of price relatives.

The stochastic approach gives a clear-cut result, but still needs empirical verification, since it does not address how efficient an estimator will be. The empirical evidence from the sampling approach confirms that, when the target for elementary aggregation is the weighted Laspeyres index, which is used for upper level aggregation in both the RPI and CPI, then the Average of Relatives is a better estimator – less biased and more efficient – than the Geometric Mean. (Even if, unusually, one has a different target weighted index, the available evidence indicates that the Average of Relatives still performs as well as or better than the Geometric Mean).

*Conclusion*

**Overall, taking account of both coverage and formula effect differences, the conclusion is that, within the limitations of how price data is collected within the UK, the RPI is as good a consumer price index as one can get for uprating purposes. The systemic differences between the RPI and the CPI are the result entirely of under-estimation by the CPI.**[78]

## 12. International Best Practice

Our discussion and assessment up to this point has been solely in terms of the statistical properties of the RPI and CPI, in the light of all the current theoretical and empirical knowledge.

It is sometimes argued, however, that we also need to look at the British RPI and CPI to see if they are consistent with "international practice" or "best international practice." Indeed, the

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[78] The qualification about the collection of price data is important, since the UK has a problem with the collection and treatment of price data for clothing, which, despite making up only about 5% of consumer expenditure, accounts for roughly half of the formula effect difference between the RPI and CPI. Improving this in line with the research conducted by the ONS would reduce the formula effect gap in clothing and would also increase the all-items CPI and decrease the all-items RPI by a noticeable amount, of the order of 0.1 percentage points in each case. c.f. the Consumer Prices Advisory Committee papers on the ONS website, summarised in the Royal Statistical Society, *Response to the National Statistician's consultation on change to Retail Prices Index*, 21 November 2012.

ONS, in its Response to the National Statistician's consultation on options for improving the Retail Prices Index goes further and identifies the only failing of the RPI as being that it "does not meet international standards," which sounds more well-defined and more demanding than merely being consistent with international practice.

On the face of it, this is a rather odd position to take. If it means that the UK must do the same as the majority of other countries, then it rules out both innovation (such as when Britain was one of the first countries to introduce annual chaining) and the use of particular practices that reflect national priorities (for example the British use of broad item definitions in order to enhance representativeness; or making the RPI non-revisable in order to improve legal certainty in its use). Provided that there are not demonstrably more efficient ways of achieving the desired ends, and provided that the calculation of the index is well-documented and transparent, there seems no statistical reason why the UK should not pursue its own priorities. Conformity may, in fact, hold intellectual dangers, since price index construction is an area that has been subject to considerable political pressure – for example from the creation of the European Monetary Union and the consequent incentive both to uniformity and to calculating an index that shows as low an inflation rate as possible.

However, if we take a wider view, there may be some merit in looking at international practice under two aspects. The first aspect refers to scientific opinion as to the best methods to use: as in most scientific areas there will be a variety of opinions and fashionable theories but it can be useful to look at what has been recommended in what circumstances. The second aspect refers to practice by national statistical authorities, not in order to conform with the majority, but to look at what they have done and why, to see if the same reasoning can with advantage be applied to Britain.

*Scientific Opinion*

Throughout this note we have discussed the different approaches taken by leading price-index statisticians. We have also quoted several times from the 2004 *Consumer Price Index Manual: Theory and Practice*, published jointly by several United Nations and other organisations, but usually credited to the ILO. We have also quoted from the 2009 *Practical Guide to Producing Consumer Price Indices*[79], also published by several international organisations and usually credited to the UN Economic Commission for Europe, which presents much the same material in a more coherent form aimed particularly at helping statistical agencies in less developed countries. As would be expected of such multi-authored, international manuals, the approach taken is not uniform and the advice given, while intended to be helpful, is seldom prescriptive. When it comes to the controversial issue of elementary aggregation, the manuals are clear that the three most common elementary aggregation formulae can all legitimately be used. While there is a marked preference by many of the authors for the Geometric Mean (based on what, to me, seems an excessive

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[79] United Nations Economic Commission for Europe et al, *Practical Guide to Producing Consumer Price Indices*, United Nations, New York and Geneva, 2009,

fondness for its algebraical elegance), using the Ratio of Averages and Average of Relatives is not only allowed but is commended in various circumstances.

In seeking a more definite indication of best practice, the one publication referred to by the UK Statistics Authority in its assessment of the RPI is a *Resolution Concerning Consumer Price Indices* passed by the Seventeenth International Conference of Labour Statisticians, meeting at Geneva in November-December 2003. This is given additional status by being included as Annex 3 of the 2004 *Consumer Price Index Manual: Theory and Practice*. This resolution covers many aspects of consumer price indices, but includes the following paragraph:

> "**43**. There are several ways in which the prices, or the price relatives, might be averaged. The three most commonly used formulae are the ratio of arithmetic mean prices (RAP), the geometric mean (GM) and the arithmetic mean of price relatives (APR). The choice of formula depends on the purpose of the index, the sample design and the mathematical properties of the formula. It is possible to use different formulae for different elementary aggregates within the same CPI. It is recommended that the GM formula be used, particularly where there is a need to reflect substitution within the elementary aggregate or where the dispersion in prices or price changes within the elementary aggregate is large. The GM has many advantages because of its mathematical properties. The RAP may be used for elementary aggregates that are homogeneous and where consumers have only limited opportunity to substitute or where substitution is not to be reflected in the index. The APR formula should be avoided in its chained form, as it is known to result in biased estimates of the elementary indices."

While allowing that all three formulas can be used, depending on circumstances, the resolution shows a preference for the Geometric Mean, based mainly on the now discredited argument from consumer substitution. It also recommends against using the Average of Relatives in its chained form. It is clear, however, from looking at the body of the *Consumer Price Index Manual: Theory and Practice*, that the reference to a bias in the chained Average of Relatives Formula applies, in practice, only to monthly chaining. Thus, in Chapter Nine:

> "**9.21** Another important property of the indices is that the Dutot and the Jevons indices are transitive, whereas the Carli is not. Transitivity means that the chained monthly indices are identical to the corresponding direct indices. This property is important in practice, because many elementary price indices are in fact calculated as chain indices which link together the month-on-month indices."

As already pointed out, this distinction between monthly and annual chaining is the same as that made by the European Commission in regulating for the composition of the HICP.

Overall, one is left with freedom of choice according to national circumstances. This was the opinion of Professor Bert Balk, of Statistics Netherlands and Erasmus University, who was one of the two renowned experts in the area of index numbers who were invited to advise or provide training to the ONS in 2012. (The other was Erwin Diewert). It should be noted that Balk had acted as referee for all the theoretical chapters of the 2004 *Consumer Price Index*

*Manual: Theory and Practice*, so that his view of the best way forward for the UK is as good an indication of international scientific opinion as any:

> **"Balk**, on the other hand, suggests that the Carli can be justified in some cases. His view is that the choice of elementary aggregate formula comes down to what we are trying to estimate with our index.
>
> He believes that if our target index is a Laspeyres type AND our elementary aggregates are heterogeneous, then it could be argued that the Carli is the most appropriate formula."[80]

*National Statistical Authorities*

The second aspect of international best practice concerns the choices made by national statistical authorities. This cannot be a matter of numbers, since the individual circumstances of countries and the political pressures they face vary so much: one needs to look at the reasons underlying their choices.

The evidence submitted by the ONS was in the form of a paper written by Bethan Evans of the ONS, *International Comparison of the Formula Effect between the CPI and RPI*, March 2012. This looked at fifteen countries that at some time over the past thirty-five years had changed elementary aggregation formula for at least part of their consumer price index. (It does not report on countries that had left their formula unchanged, usually as the Ratio of Averages, which remains the most widely used formula internationally).

In five cases the switch was from the Ratio of Averages to the Geometric Mean, sometimes only for a few items, because of the introduction of more heterogeneous elementary aggregates or worries about the influence of higher priced products within items.[81] This is not relevant to the RPI, which uses the Ratio of Averages only for homogeneous items.

In the case of Luxembourg the reason given for switching from the Average of Relatives was Eurostat preference, i.e. it was aligning its national consumer price index towards the HICP, which, again, is not relevant to the RPI.

In only two cases – Canada in an earlier switch from the Average of Relatives to the Ratio of Averages in 1978; and Denmark in switching some of its items from the Average of Relatives to the Geometric Mean in 2000, was "price bouncing" when using the Average of Relatives given as a reason for the switch. This might have been seen as an example for the UK to follow, since lack of transitivity in the Average of Relatives was the one statistical reason given by the ONS for a switch to the Geometric Mean. However, as already pointed out, Canada and Denmark are unusual in using monthly chaining, so that, although these changes might have been sensible for them, their example is of little relevance to the UK.

In eight cases the reason given for switching at least some elementary aggregation from the Average of Relatives or the Ratio of Averages to the Geometric Mean was consumer

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[80] ONS, *Managing the Formula Effect*, presentation to the Royal Statistical Society RPI-CPI User Group, June 2012, slide 51.

[81] The entry in the ONS paper for Ireland is hard to understand, since in 1997 it switched some of its items from the Ratio of Averages to the Geometric Mean, with Axiomatic (transitivity) given as the reason, whereas both of these are transitive formulae.

substitution. Moreover, more of these were Anglophone countries like Australia, Canada (the 1995 switch), USA, New Zealand, rather than EU countries where the political desire to align with the HICP influenced the statistical decisions. Had their reason for switching been valid, this would have meant that the UK should take a serious look at following their example. However, as we have seen, the ONS now accepts that consumer substitution is not even a weak reason for preferring the Geometric Mean, so that the example of these countries is not normative. It should also be pointed out that in the case of the USA, whose Boskin Commission report was very influential in other countries as well, an explicit motive for changing from a type of Average of Relatives to the Geometric Mean for 61% of its elementary aggregates, was a desire to move from a cost-of-goods index to a constant-utility index (what they call a cost-of-living index). And the ONS still aims to keep the RPI (and, indeed, the CPI) as a cost-of-goods index,[82,83] The ONS ought, therefore, to accept the logic of its position that these countries were mistaken in their reason for switching. Even so, the fact that they *have* switched means that the UK is almost alone in still using the Average of Relatives for its consumer price index[84], and the loneliness of that position, even if fully justified, may be the true cause for the unease felt by the ONS. There is no reason for such unease. The UK's use of wide item definitions means that the underestimate from using the Geometric Mean for the RPI would be unacceptably high for Britain, much higher than the 0.1 to 0.2 percentage points experienced by these other countries when they switched to the Geometric Mean.[85] It should also be noted that many countries use the Average of Relatives for elementary aggregation in their other price indices – more commonly than they use of the Geometric Mean – suggesting that a mistaken attempt to allow for consumer price substitution has been the main driver for a different treatment of consumer price indices.[86]

Overall, therefore, historical formula switching by other countries does not establish a pattern of best practice, nor is it relevant for the UK.

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[82] Boskin et al., Final Report of the Commission to Study the Consumer Price Index, US Senate, Committee on Finance, 1996, available at http://www.ssa.gov/history/reports/boskinrpt.html#cpi6, Executive Summary, paragraph 4.

[83] See also the critique of the Boskin Commission Report by the Bureau of Labor Statistics U.S. Department of Labor "Measurement issues in the consumer price index". *Statistical Journal of the UN Economic Commission for Europe*, 1998, Vol. 15, Issue 1.

[84] The only other OECD country to use a form of the Average of Relatives for first-stage aggregation in its consumer prices index is Israel. c.f. OECD Main Economic Indicators, available at http://stats.oecd.org/mei/default.asp?lang=e&subject=8; M. Sabag and Y. Finkel, "The Israeli Consumer price index: alternative computing methods for an 'Item Price Index'", *Statistical Journal of the United Nations ECE*, vol 11, 1994, pp 95-118.

[85] Bethan Evans, op cit.

[86] c.f. OECD, *Producer Price Indices - Comparative Methodological Analysis*, 2011, Table 9, which shows that 16 of the OECD countries used the Average of Relatives, 11 used the Geometric Mean and 5 used the Ratio of Averages. On the use only of the Ratio of Averages and Average of Relatives in all German price statistics see https://www.destatis.de/DE/ZahlenFakten/GesamtwirtschaftUmwelt/Preise/Grosshandelspreisindex/Methoden/ Grosshandelsverkaufspreise.html

## 13. The RPI's status as an Official Statistic and not a National Statistic

*Definition of a National Statistic*

The UK Statistics Authority was established in April 2008 as a board responsible for promoting and safeguarding the production and publication of official statistics. In April 2009 it published a Code of Practice for Official Statistics, which elaborated eight Principles:

Principle 1: Meeting user needs
Principle 2: Impartiality and objectivity
Principle 3: Integrity
Principle 4: Sound methods and assured quality
Principle 5: Confidentiality
Principle 6: Proportionate burden
Principle 7: Resources
Principle 8: Frankness and Accessibility

Official Statistics that the UK Statistics Authority considers compliant with the Code of Practice for Official Statistics are designated National Statistics.

*Background to the RPI's change of status*

In December 2010 the UK Statistics Authority published an assessment of all the consumer price statistics produced by the ONS, including both CPI and RPI.[87] In accordance with its standard wording, it concluded that:

> "The Statistics Authority judges that the statistics covered by this report are readily accessible, produced according to sound methods and managed impartially and objectively in the public interest, subject to any points for action in this report."

The points for action dealt mainly with greater openness about release dates and times and the publication of more information about the statistics. They included:

> "**Requirement 3** Publish information about the history and the reasons for the differences in scope and methods between the CPI and RPI; and explain the implications that these differences have for the uses to which these statistics are put."

In parallel with its Assessment Report, the UK Statistics Authority published a long analysis of the use and communication of consumer price statistics in the UK, including, in Annex 6 a detailed discussion of international practice, which was available to guide its assessment. [88]

Following the requirements and suggestions of the UK Statistics Authority, the ONS initiated a programme of research into the uses of the CPI and RPI for macroeconomic purposes and as unprating indices and how both indices might be improved. In accordance with Requirement 3, the ONS in October 2011 published a note, *The Implications of the Differences between the Consumer Prices index and the Retail Prices Index*. This provided an even-handed treatment of the two indices, concluding that they both had strengths and

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[87] UK Statistics Authority, Assessment Report 97, *Consumer Price Indices (produced by the Office for National Statistics)*, December 2010.
[88] UK Statistics Authority, "Communicating Inflation," *Monitoring Brief /2010*, 6 December 2010, Appendix 6

weaknesses and that the choice between them for macroeconomic or uprating purposes was mainly one of coverage, but that the aggregation formula was also an issue, with something to be said on either side, depending on how prevalent consumer substitution was and whether it should be taken account of at all. Accordingly, on 31 January 2012, the UK Statistics Authority confirmed the RPI as a National Statistic.[89]

One strand of the ONS research programme research focused on introducing owner-occupied housing within the CPI. This will be discussed separately. Another strand of research focused on explaining the formula effect difference between the CPI and RPI and how elementary aggregation might be improved in both indices. It looked particularly at clothing, which, although representing only about 5% of consumer expenditure, gave rise to half the formula effect difference. By July 2012 this research had produced promising results such as tightening item definitions, making greater use of seasonal adjustment, using outlet stratification and changing the base year from the sale-affected January.[90] All or most of these changes could have been introduced at the beginning of 2014. Taken together, they would have improved the accuracy of both indices, and would have approximately halved the formula effect gap in clothing and reduced the overall formula effect between RPI and CPI.[91]

However, at its meeting on 13 September 2012, CPAC, the Consumer Prices Advisory Committee[92], agreed that the National Statistician should consult only on changes to be introduced to the elementary aggregation formula for the RPI, with any changes to be introduced from March 2013. It is not clear whether this change of direction originated from within the ONS, the UK Statistics Authority or from other members of CPAC, which, while including private sector members, was dominated by officials from the UK Statistics Authority, ONS, HM Treasury and the Bank of England.

The consultation document was published on 8 October 2012, with consultation closing on 30 November 2012. It set out four options. One was for no change – which the consultation argument argued against – and the other three involved replacing the Average of Relatives within the RPI, either entirely or just for clothing.

The consultation drew a large response, which was overwhelmingly for no change. 332 responses favoured no change; 9 favoured changes only in clothing; 11 favoured replacing the Average of Relatives throughout the RPI, though not necessarily with the Geometric Mean.

Of the responses with a statistical content, which included from the Royal Statistical Society and from expert statisticians, 44 favoured no change; 4 favoured changes only in clothing; 2

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[89] UK Statistics Authority, letter from Richard Alldritt, Director General, Monitoring and Assessment, to Stephen Penneck, Director General of the Office of National Statistics, 31 January 2012.

[90] Switching from a January to a December base month may, in any case, be necessary if the CPI is to comply with HICP requirements.

[91] c.f. the Consumer Prices Advisory Committee papers on the ONS website, summarised in the Royal Statistical Society, *Response to the National Statistician's consultation on change to Retail Prices Index*, 21 November 2012.

[92] CPAC met for the last time in January 2013 and was disbanded in February 2014, to be replaced by two new advisory panels. c.f. UK Statistics Authority, Review of the Governance of Price Statistics, February 2014.

favoured replacing the Average of Relatives throughout the RPI by the Ratio of Averages; 3 favoured replacing the Average of Relatives throughout the RPI by the Geometric Mean.[93]

Faced with such an overwhelming response, the National Statistician decided to retain the existing elementary aggregation formulae in the RPI. However, she announced that in future the RPI would be subject to only routine updating. She also announced the creation of a new index, RPIJ, which, in line with the ONS' originally favoured option, replaces the Average of Relatives with the Geometric Mean throughout the RPI.

In its response to the consultation, the ONS acknowledged the statistical arguments put to it. In particular, it conceded that it was unable to provide a numerical estimate of the size of "price bouncing" – which was the sole statistical argument it had put forward in favour of abandoning the Average of Relative – since any effect would be indistinguishable within the chain drift that affects both RPI and CPI. However the reason the National Statistician gave for retaining the current RPI was only that, "there is significant value to users in maintaining the continuity of the existing RPI's long time series without major change, so that it may continue to be used for long-term indexation and for index-linked gilts and bonds in accordance with user expectations." The need to maintain the integrity of contracts involving the RPI and avoid the prospect of widespread litigation was also the reason for retaining its name (and giving a new name to RPIJ, rather than the other way round, as had been desired by the Consumer Prices Advisory Committee[94]).

*Change of Status of the RPI*

On the same date as the ONS statement, on 10 January, the Board of the UK Statistics Authority announced that it would be undertaking a re-assessment of the RPI as to whether it still qualified as a National Statistic.

On 14 March 2013, the UK Statistics Authority duly published a short assessment, based solely on material supplied to it by the ONS, including the responses to the consultation carried out by the ONS on improving the RPI. Contrary to its usual practice, the UK Statistics Authority did not conduct its own written consultation, nor consult outside experts. It concluded that National Statistics designation should be withdrawn from the RPI and its derivative indices for the following two reasons:

> "i) the finding that the methods used to produce the RPI are not consistent with internationally recognised best practices (para 3.4); and
>
> ii) the decision to freeze the methods used to produce the RPI, and only to contemplate 'routine' changes (para 3.5)."

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[93] Denise Osborn, Professor of Econometrics at Manchester University, was asked by the ONS asked to advise them on the statistical responses received in the consultation. Her conclusion was: "The consultation responses made available to me provide a number of coherent arguments in support of retention of Carli aggregation at the elementary level within [the RPI], at least until further work is undertaken within ONS." Denise Osborn, *Comments on Responses to ONS Consultation on Improving RPI*, 19 December 2012, available at http://www.ons.gov.uk/ons/about-ons/get-involved/consultations/archived-consultations/2012/national-statistician-s-consultation-on-options-for-improving-the-retail-prices-index/index.html.

[94] ONS, *Summary of the seventeenth meeting of the Consumer Prices Advisory Committee (CPAC)*, January 2013, p 2.

Paragraphs 3.4 and 3.5 are the only two analytical paragraphs in the assessment covering the point at issue, namely "sound methods and assured quality." In relation to internationally recognised best practice, the assessment refers only to two documents, the 2003 Labour Statisticians' resolution and the March 2012 note by the ONS on *International Comparison of the Formula Effect between the CPI and RPI*. As we have seen, when taken in context – which may not have been available to the Statistics Authority – neither indicates any deviation by the UK from international best practice. Moreover, over the two years since the UK Statistics Authority's favourable December 2010 assessment of the RPI, new research, both within and outside the UK, pointed not towards greater support of the Geometric Mean but rather towards greater scepticism about its use. The UK Statistics Authority's assessment team seemed aware of these limitations, since the conclusion of paragraph 3.4 is carefully worded: "On the basis of the evidence presented by the National Statistician, the Assessment team therefore considers that the RPI does not comply with the *Code*."

The operative reason for the UK Statistics Authority's decision was, therefore, that expressed in paragraph 3.5, "The Assessment team considers that the decision [by the ONS] to effectively freeze the formula used at the elementary aggregate level in the RPI, and contemplate only 'routine' changes is inconsistent with the requirement in the *Code* to seek to achieve continuous improvement." In other words, the loss of National Statistic designation was only because the ONS said in January 2013 that in future it would not do more than routine updating of the RPI.

Unusually, and in apparent contravention of the UK Statistics Authority's *Criteria for not awarding the National Statistics designation*,[95] the assessment of the RPI did not set out steps which, when completed, would allow the designation to be awarded. Perhaps the UK Statistics Authority felt itself unable to come with an independent conclusion about the weaknesses in the RPI or with recommendations about how it could be continuously improved while still meeting the needs of users for a fixed-basket index (i.e. one which does not use the Geometric Mean for elementary aggregation), even though there were copious suggestions for such improvement in the consultation responses. Hence its simple agreement with the then National Statistician that no more than routine updating should take place.

One is left, therefore, with no valid reason for the withdrawal of National Statistic designation from the RPI, other than the self-imposed decision by the National Statistician not to undertake other than routine updating. It is to be hoped that the ONS consultation on consumer price statistics announced on 15 June 2015[96] will be able to point a way towards keeping the RPI in a form that will meet users' needs while making sure that it retains its existing statistical quality into the distant future.

In any case, one should not regard the National Statistic designation as a gold standard or guarantee of accuracy. A glance at the UK Statistics Authority's Assessment Reports shows that many much-used and well-regarded statistics are refused designation unless and until

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[95] UK Statistics Authority, *Criteria for not awarding the National Statistics designation*, 23 September 2009 states: "In any case, where the National Statistics designation is not awarded or is cancelled, the Assessment report will set out steps that when completed will result in the designation being awarded without a further Assessment being conducted."

[96] UK Statistics Authority, *Measuring Consumer Prices: the Options for Change*, June 2015.

they can meet a sometimes very extensive list of recommendations: the 12 June 2014 Assessment of DECC's statistics on Energy and Climate Change is just one example. In the circumstances, the failure of the RPI to retain designation as a National Statistic should be regarded as the outcome of organisational imperatives, rather than any comment on its current accuracy.

# 14. Housing, CPIH and RPIJ

Expenditure on owner-occupied housing is an important part of UK consumer spending and there is general agreement that it should be included in any all-items consumer price index. However, because housing is a long-lasting asset, its treatment in a consumer price index is not straightforward. There are three main approaches that have been taken.

*Payments Approach*

This approach looks at the expenditures actually made by consumers in occupying their own houses, and most closely follows the traditional approach to the construction of consumer price indices. It is generally regarded as the most suitable for consumer price indices that will be used for the adjustment of compensation or income.[97] It requires good data on the housing stock and on mortgage interest payments, but is conceptually straightforward. This is the approach that has been used by the RPI since 1975 and was described in the first section of this note. Because mortgage interest payments move in the same direction as interest rates, it is usually considered undesirable that they be included in a monetary policy indicator, particularly in a country like the UK which uses mainly floating interest rates for its mortgages, (because an increase in interest rates, which in the longer term is meant to damp down inflation, will give rise to an immediate short-term increase in measured inflation through increasing mortgage interest payments). That is why, since 1975 the UK has published an adjusted measure, RPIX, which excludes mortgage interest payments, and which served as the monetary policy indicator until 2003.

In the UK the debate over owner-occupied housing has been conducted in terms of what would be suitable for inclusion in the CPI, in its primary role as a macroeconomic indicator[98]. The payments approach has continued to be seen as appropriate for the RPI, considered as an uprating index,[99] but the ONS never considered it for the CPI, since it saw the inclusion of interest payments as incompatible with the underlying HICP philosophy.[100]

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[97] UN ECE, *Practical Guide to Producing Consumer Price Indices*, Chapter 9; ILO, *Consumer Price Indices Manual*, Chapter 10 and Annex 3 paragraph 15.
[98] CPAC(10)15, *Progressing the implementation of owner occupiers' housing costs in the Consumer Price Index*, September 2010.
[99] CPAC(09)03, *Measurement of Housing in the Consumer Prices Index*, July 2009.
[100] There is also the point that the depreciation element of owner-occupied housing expenditure is an imputed rather than an actual cost, and this, too, goes against the HICP philosophy. However, since the ONS eventually settled for the rental equivalence measure of owner-occupiers' housing cost in CPIH, having an imputed cost should not have been a significant obstacle.

*Net Acquisitions Approach*

This approach treats houses like any other consumer good[101], counting the cost *to the household sector*, of buying them from another sector (e.g. housebuilders or developers) or building them themselves (self-build). It would also include the cost of house extensions, house maintenance and insurance and property taxes, but would not include the cost of credit. It is a *net* acquisitions approach, because it excludes the purchase of second-hand houses within the household sector. It is, therefore, a measure that is sensitive to new house prices and also to the volume of new house construction (since that affects the weight with which owner-occupied housing enters the overall consumer price index) but it is not sensitive to mortgage interest rates.

Because it is based on actual monetary expenditure on a consumer good, the net acquisitions approach fits in well with the requirements of a monetary indicator and it is the measure that Eurostat, in consultation with national statistical authorities, has chosen for eventual inclusion in the HICP. National statistical authorities in EU countries will be required to provide a free-standing owner-occupied house price index on this basis from 2015.

This was one of the measures that the ONS consulted on in 2012 for inclusion in CPIH, although it was not its preferred or eventually chosen measure.

*Narrow User Cost Approach*

This approach follows most closely the national accounts principles, and aims to measure to the cost to home owners of the consumption benefit that they derive from their houses, which are treated as capital assets. It has the advantage of being comprehensive (treating all house owners equally) but the disadvantage that much of the cost must be imputed or estimated and cannot be directly observed. It proceeds by valuing the whole housing stock and then asking what is the cost to the homeowners of tying up their money in bricks and mortar rather than investing in financial assets. This is quite a difficult concept, particularly if it means trying to take capital gains into account. The approach considered by the ONS was to use the *real rate of interest*, i.e. the actual rate of interest (mortgage interest payments for the proportion of the housing stock that is mortgaged, a low-risk financial asset for the rest) minus inflation. Then, other housing costs such as depreciation, maintenance, insurance and property taxes are added on top.

The ONS originally considered this as one option for inclusion in the CPIH, but rejected it because of the subjective judgement involved in choosing an appropriate real interest rate, even if real interest rate volatility was reduced by using a long run moving average rate. Instead, the ONS chose a simpler (and quite widely-used) variant, namely the rental equivalence approach, which was introduced from the beginning of 2013, even though it was supported by only a minority of consultees.

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[101] There is a qualification in that land should be treated as a capital asset and not a consumer good and so ought to be excluded from the house price. This is not much of a problem for some countries, like Australia, where self-build or project houses are the norm, or for countries where the cost of land makes up a relatively small proportion of a house's total cost.

*User Cost Variant: the Rental Equivalence Approach*

This variant of the user cost approach measures the consumer services provided to homeowners by assuming that they are renting their houses from themselves, and looking at the market rate they would pay for it. This concept will be familiar to older readers from the Schedule A income tax that owner-occupiers had to pay up until 1963 (the *quid pro quo* of which was that they were entitled to tax relief on maintenance expenditure and on mortgage interest payments)[102]. Experience with Schedule A points to one difficulty with this approach, namely finding an appropriate rental comparison. The sample previously used for private rentals was deemed to be too narrow and not well matched with owner-occupied housing, and instead private housing rental data from the Valuations Office Agency (VOA) plus comparable data from the Welsh and Scottish Governments was used. However, problems with this data have caused the CPIH series to lose its National Statistic designation and to be reclassified in August 2014 as an experimental series. There is also a conceptual decision to be made as to whether maintenance payments by the owner-occupier are to be included in the imputed rental or included separately, (they were moved into the imputed rental in 2014) and, in the case of council tax, whether it should be included at all (included from 2014).

A second problem with this approach is that, over the short and medium term, private sector rentals move differently from house prices and thus the rental equivalence approach bears a tenuous relationship to the costs that home owners actually face. In fact over the period 1989 to 2010 there was a marked negative correlation between them.[103]
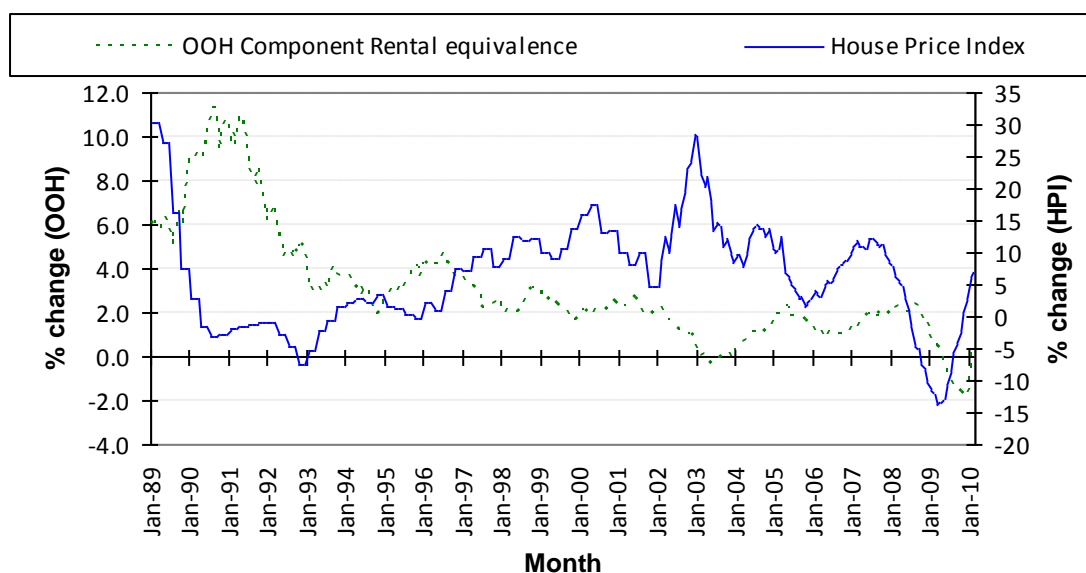


Because the rentals series is relatively smooth and has increased only slightly faster than the CPI itself, CPIH and CPI have moved very closely together. Before the 2008 recession, CPI increased slightly faster than CPI, since then slightly more slowly.

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[102] Margaret Thatcher MP, *Do you qualify for a refund?*, Finchley Times, 18 March 1960. The 1936 rental values were still being used, not having been revalued since then.
[103] CPAC(10)15, Annex B
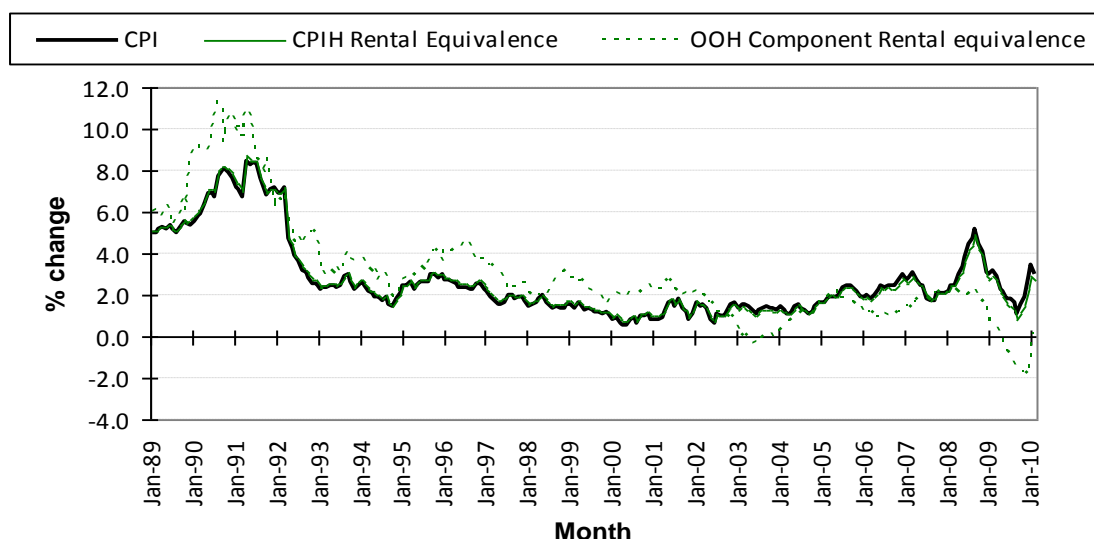
It remains to be seen how well CPIH will perform once its data problems have been alleviated. It has not yet achieved credibility as a monetary policy indicator[104] and is unlikely ever to be seen as a realistic measure of the costs faced by owner-occupiers – see the detailed arguments in the response by the RPI CPI User Group to the ONS consultation on owner occupier housing costs, in which they justify their opening remarks:[105]

> "The User Group overall considers that the recommendation of Rental Equivalence is not appropriate. In ONS's own words, this is a proxy for the actual costs faced by owner occupiers. It is a proxy which we find neither convincing in concept nor, on the evidence available, in practice."

*RPIJ*

As we have seen, the ONS, following overwhelming opposition to their plan to replace the Average of Relatives in the RPI, decided in January 2103 to retain the existing formulation of the RPI. However, at the same time they announced that they were going ahead with a new index, called RPIJ, which would carry out their favoured option of replacing the Average of Relatives, wherever it occurred in the RPI, by the Geometric Mean. This option had been supported by 2 of the 406 consultation responses the ONS had received, including 1 of the 44 statistical responses.

Although the RPIJ still uses the Ratio of Averages for the homogeneous items where it is used in the RPI, these cover groups which are responsible for very little of the formula difference between the CPI and RPI. Thus, changing the Average of Relatives to the Geometric Mean effectively makes RPIJ, despite its name, behave like the CPI. It does however, include the owner-occupier housing measures and the other coverage differences used in the RPI, albeit sometimes aggregated using the Geometric Mean. It is thus best seen as an adaptation of the CPI using the payments approach to owner-occupier housing rather

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[104] In the last four issues of the fifty-page quarterly Bank of England *Inflation Report* CPIH is referred to only three times, including in footnotes.
[105] Downloadable from the page http://www.ons.gov.uk/ons/about-ons/get-involved/consultations/archived-consultations/2012/owner-occupiers-housing-costs/index.html

than the rental equivalence approach. As such, it includes a sensitivity to both house prices and mortgage interest payments that CPIH does not have. It is thus likely to fall between two stools: still being unacceptable as an uprating index because its use of the Geometric Mean means that it is no longer a fixed-basket index and is underestimating inflation by about 0.65 percentage points per annum; and unacceptable as a monetary policy indicator because of its sensitivity to movements in the mortgage interest rate.

## 15. Tax and Price Indices and the Croner Reward Index

There are a number of indices produced by the ONS that either include direct taxes (not normally counted as a consumption expenditure) or exclude indirect taxes like VAT (which are included in the price of consumer goods).

The indices that exclude indirect taxes are RPIY (RPI excluding mortgage interest payments and indirect taxes), CPIY (CPI excluding indirect taxes), CPI-CT (CPI at constant tax rates). These have a use in macroeconomic analysis, because they can give an indication of underlying inflation, but are unsuitable for uprating purposes and will not be considered further.

The Tax and Price Index, TPI, based on the RPI, is more interesting. It was introduced in 1979 under the Conservative administration of Margaret Thatcher, when the government had a policy of increasing indirect taxes and decreasing direct taxes. Increased VAT showed up immediately in higher prices, indicating that people were worse off, so the government wanted to show that this was offset, to some extent, by lower direct taxes. (Of course, this obscured the distributional aspects, since indirect and direct taxes affected different groups of people differently). After its brief moment in the limelight the TPI has languished in obscurity ever since, perhaps because, with the development of stealth taxes and reliance on real fiscal drag, governments have preferred to emphasise falling direct tax rates rather than a steady direct tax burden. But the message is not entirely gloomy: from 1983 to 2003 TPI inflation was generally slightly below RPI inflation. Since then it has marched remarkably closely in step with RPI inflation up until 2012, after which the "taxless recovery" has seen it once more grow slightly more slowly. In a world in which direct taxes, tax credits and benefits are closely intertwined, there are ambiguities about what the TPI measures and the changing distribution of the tax burden also makes any reference to it for uprating purposes problematic – it is probably best viewed as an ancillary macroeconomic indicator.

**Tax and Price Index and Retail Prices Index, 12 month inflation rates**
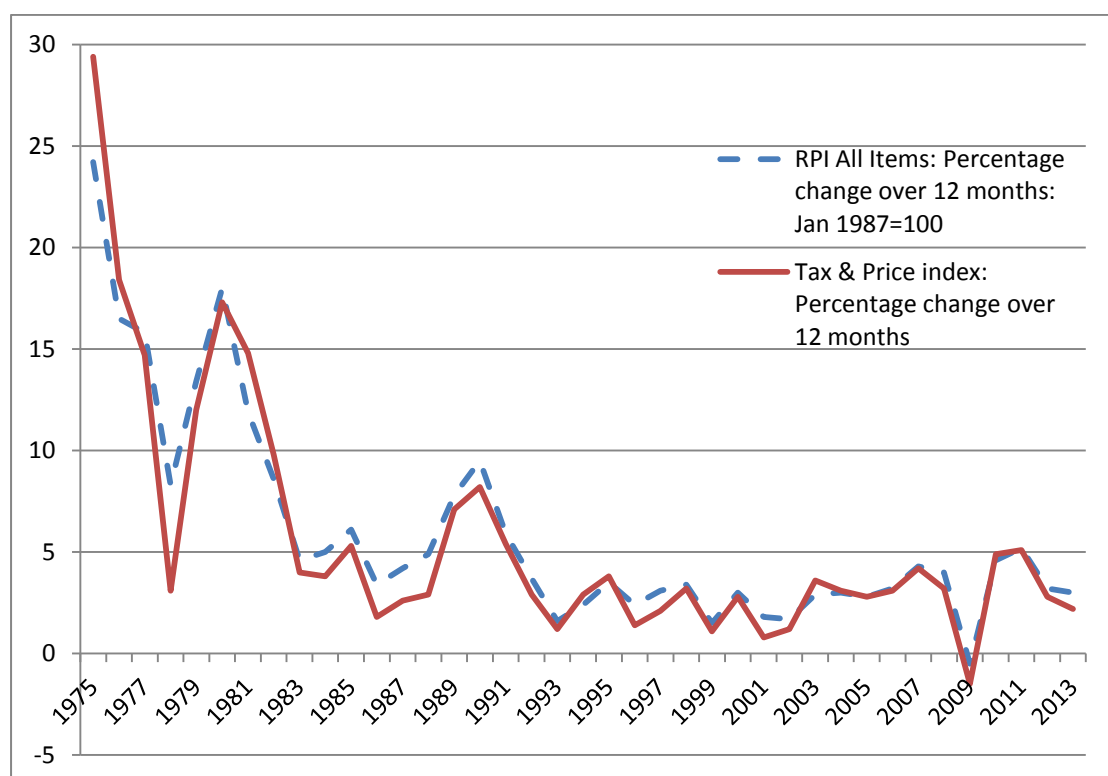


A somewhat similar index is compiled by Croner Reward, a private-sector Human Resources company, and in this case the index is designed to be applicable for pay comparison purposes. Croner Reward uses its own regionally-differentiated price samples, taking a very much smaller sample than the ONS (except for the problematic clothing sector, where, just when an independent approach would be illuminating, it uses the RPI data). In compiling the regional and national price indices, it uses the expenditure patterns for eight different representative households (standards of living), with the expenditure patterns for these households being based on the Family Expenditure Survey data. These patterns seem not to have been revised since 1982, so it is a direct rather than a chained index. However, for housing it makes a different assumption: its lowest standard household is assumed to be renting a three-bedroomed semi-detached council house; all the other standards are assumed to be buying a house of an appropriate size with a mortgage varying between 30% and 68% according to the standard. When aggregating the different standards together, it uses weights based on the population in each standard.

Besides calculating a consumer price index on this basis, Croner Reward also calculates a Required Income index, which adds in a savings requirement (rising from 0.75% of expenditure for Standard A to 5.4% for Standard F), national insurance, contributory pension payments and income tax. This is not the same as, but can be compared to, the Office for National Statistics' Tax and Price Index.

Croner Reward's housing assumptions are appropriate for its purposes, which are to measure the living costs of workers who are assumed to be mobile between regions and who will need

to be paid salaries to enable them to move to a new job.  It does mean that housing has a higher weight in its consumer price index than in the RPI – about 32%, compared to 25% in the RPI, and, within housing, both house prices and mortgage interest payments have a high weighting.  This is reflected in the movements of the Croner Rewards indices, relative to the most nearly comparable ONS indices.

## 16.  Pensioner Indices, the Rossi Index and the Personal Inflation Calculator

*Pensioner Indices*

The ONS publishes price indices, based on the RPI, designed to match the spending patterns of one-person and two-person pensioner households, for pensioners excluded from the standard RPI, based on the Living Costs and Food Survey data.  It is thus not representative of all pensioners.  The indices exclude some categories of expenditure, e.g. canteen meals and child-minding fees, and have different weights, e.g. a higher weight for food.  However, the main difference is the exclusion of almost all housing costs.  This is partly on the assumption that pensioners will be shielded to some extent from changes in housing costs by e.g. council tax support (formerly council tax benefit), partly because, if they are owner-occupiers, they are more likely than average to own outright, without a mortgage.  But the ONS also points to the difficulty of establishing a housing expenditure pattern for pensioners as a group.

*The Rossi Index*

The ONS also publishes the Rossi index (named after a former Conservative Minister of State for Social Security), which up until 2011 was used for uprating *income-related* State benefits. It is the all-items RPI excluding mortgage interest payments, housing depreciation, rents and council tax – i.e. excluding almost all housing expenditure other than maintenance and utility bills (other State benefits were uprated in line with the all-items RPI).

Inflation according to the Rossi index and the single-pensioner household index is shown in the chart below, compared to all-items RPI inflation (the two-person pensioner household inflation was very similar to that for one-person households, and is excluded for the sake of clarity).  It shows a long period up to 2007 when pensioner inflation was generally a little below RPI inflation, whereas since then it has been above it.  This is caused mainly by the exclusion of housing.

**Pensioner, Rossi and RPI 12 month inflation rates**
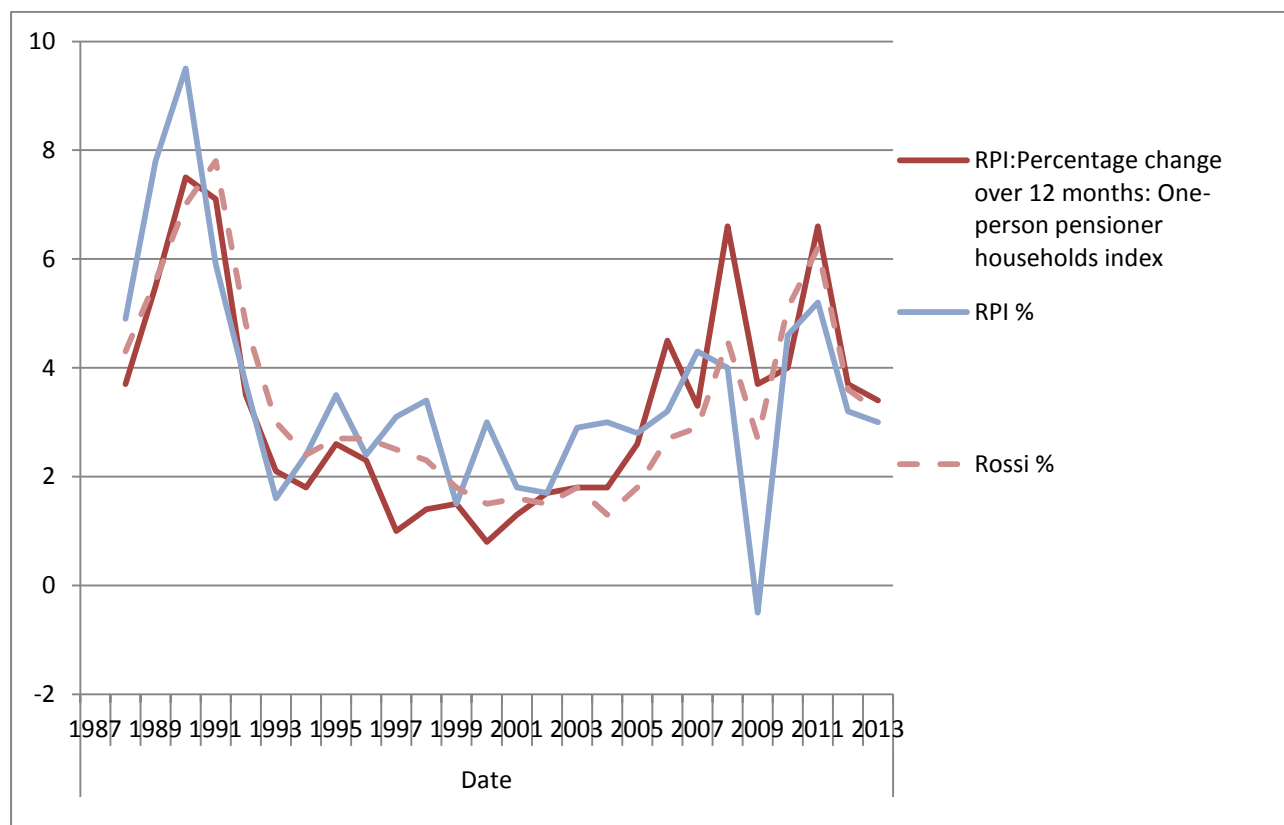


*Personal Inflation Calculator*

The ONS also provides – for those using the right sort of web browser – an on-line personal inflation calculator based on RPI inflation rates for the 23 main categories of spending, including mortgage interest, rent, car expenditure and fuel.  Users can input their own estimated expenditure on the various categories and get an estimate of the inflation rate that applies to them.  This is one service where the coverage basis of the RPI is essential, since the calculation of personal inflation requires inputting actual housing expenditure.

## Acknowledgements

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# Appendix 1

**Price Indices in an Economy with Supply and Demand Shocks, Two Goods and a Representative Consumer and Producer**

Assume a Cobb-Douglas utility function: $U = x^\alpha y^{1-\alpha}$

By taking a total differential and setting $dU = 0$ we get a family of indifference curves:

$$\frac{dy}{dx} = -\frac{\propto}{1-\propto}\frac{y}{x}$$

When the consumer is in equilibrium, the slope of the indifference curve = ratio of relative prices:

$$-\frac{P_x}{P_y} = \frac{dy}{dx} = -\frac{\propto}{1-\propto}\frac{y}{x}$$

which can be written: $\qquad\qquad \frac{P_x x}{P_y y} = \frac{\alpha}{1-\alpha}$ $\qquad\qquad\qquad$ *

Assume an elliptical production possibility frontier: $\theta x^2 + y^2 = k^2$

Differentiating, the slope at any point on the production possibility frontier is given by:

$$\frac{dy}{dx} = -\theta\frac{x}{y} \quad \text{which, at the supplier's equilibrium,} = -\frac{P_x}{P_y}$$

In a market equilibrium between supply and demand, the price ratio faced by consumer and supplier is equal:

$$\therefore \ \theta\frac{x}{y} = \frac{\alpha}{1-\alpha}\frac{y}{x}$$

$$\therefore \quad x = y\sqrt{\frac{\alpha}{1-\alpha}}$$

Substituting this result in the equation for the production possibility frontier gives, in equilibrium:

$$\theta y^2\frac{\alpha}{1-\alpha} + y^2 = k^2$$

$$\therefore \quad y = \sqrt{1-\propto}\ k$$

$$\text{and } x = \sqrt{\frac{\propto}{\theta}}k$$

Now introduce the consumer's income constraint: she allocates her income *m* between the two goods:

$$P_x\ x + P_y\ y = m$$

Using the unit elasticity expenditure property of the Cobb-Douglas, established at $*$ above:

$$P_x \, x = \alpha \, m$$

and $\;\; P_y \, y = (1 - \alpha) \, m$

Substituting in the values already calculated for *x* and *y*:

$$P_x = \sqrt{\alpha\theta} \; \frac{m}{k}$$

$$P_y = \sqrt{1 - \alpha} \; \frac{m}{k}$$

Now, introduce a supply shock, altering the production possibility frontier to: $\;\; \varphi x^2 + y^2 = k^2$

The prices and quantities in the initial and subsequent periods are set out in the table below:

Table 1 (Appendix 1): Price and Quantities before and after a Supply Shock

|  | $P_x$ | x | $P_y$ | y |
|---|---|---|---|---|
| period 0 | $\sqrt{\alpha\theta} \; \dfrac{m}{k}$ | $\sqrt{\dfrac{\alpha}{\theta}} \; k$ | $\sqrt{1-\alpha} \; \dfrac{m}{k}$ | $\sqrt{1-\alpha} \; k$ |
| period 1 | $\sqrt{\alpha\varphi} \; \dfrac{m}{k}$ | $\sqrt{\dfrac{\alpha}{\varphi}} \; k$ | $\sqrt{1-\alpha} \; \dfrac{m}{k}$ | $\sqrt{1-\alpha} \; k$ |

A weighted Laspeyres price index, calculated as $\;\; \dfrac{P_{x,1}}{P_{x,0}} \left( \dfrac{P_{x,0} \, x_0}{m} \right) + \dfrac{P_{y,1}}{P_{y,0}} \left( \dfrac{P_{y,0} \, y_0}{m} \right)$

$$= \; \frac{P_{x,1}}{P_{x,0}} \, \alpha + \frac{P_{y,1}}{P_{y,0}} \, (1 - \alpha)$$

$$= \; \sqrt{\frac{\varphi}{\theta}} \, \alpha + (1 - \alpha)$$

The weighted Paasche index is $\qquad \dfrac{P_{x,1} \, x_1 + P_{y,1} y_1}{P_{x,0} \, x_1 + P_{y,0} y_1}$

$$= \; \frac{1}{\sqrt{\frac{\theta}{\varphi}} \, \alpha + (1 - \alpha)}$$

The weighted Geometric index is calculated as $\qquad \left( \dfrac{P_{x,1}}{P_{x,0}} \right)^{\alpha} \left( \dfrac{P_{y,1}}{P_{y,0}} \right)^{1-\alpha}$

$$= \; \left( \frac{\varphi}{\theta} \right)^{\frac{\alpha}{2}}$$

Fisher's Ideal index is the geometric mean of the Laspeyres and Paasche indices: $\sqrt{\dfrac{\varphi\alpha+(1-\alpha)\sqrt{\theta\varphi}}{\theta\alpha+(1-\alpha)\sqrt{\theta\varphi}}}$

A straightforward comparison shows that the Laspeyres index will always be greater than the Paasche index (equal when $\theta = \varphi$), with the Fisher's Ideal index between them.

The Geometric Mean index will equal the Laspeyres index in the trivial case where $\theta = \varphi$. Otherwise, it will always be below it. To see this, consider both indices as a function of $A = \sqrt{\dfrac{\varphi}{\theta}}$. The derivative of the Laspeyres index with respect to $A$ is a constant, $\alpha$. Likewise, the derivative of the Geometric Mean index is a diminishing function of $A$. The two derivatives are equal only where $\theta = \varphi$, when the values of the two indices also coincide. Therefore, for all values of $A$, the value of the Geometric Mean index is below that of the Laspeyres index.

The Geometric Mean index will be equal to the Fisher's Ideal index when $\alpha = ½$ (i.e. equal expenditure on the two goods), but will diverge from it as $\alpha$ goes towards 0 or 1.

Now, instead, introduce a demand shift, with the Utility function changing to $U = x^{\beta}\,y^{1-\beta}$, with unchanged income, $m$. The initial and subsequent period price and quantities will be:

Table 2 (Appendix 1): Prices and Quantities before and after a Demand Shock

|  | $P_x$ | x | $P_y$ | y |
|---|---|---|---|---|
| period 0 | $\sqrt{\alpha\theta}\,\dfrac{m}{k}$ | $\sqrt{\dfrac{\alpha}{\theta}}\,k$ | $\sqrt{1-\alpha}\,\dfrac{m}{k}$ | $\sqrt{1-\alpha}\,k$ |
| period 1 | $\sqrt{\beta\theta}\,\dfrac{m}{k}$ | $\sqrt{\dfrac{\beta}{\theta}}\,k$ | $\sqrt{1-\beta}\,\dfrac{m}{k}$ | $\sqrt{1-\beta}\,k$ |

The Laspeyres price index will be: $\sqrt{\dfrac{\beta}{\alpha}}\,\alpha + \sqrt{\dfrac{1-\beta}{1-\alpha}}\,(1-\alpha)$

$$= \sqrt{\alpha\beta} + \sqrt{(1-\alpha)(1-\beta)}$$

The Paasche index is: $\dfrac{m}{\sqrt{\alpha\theta}\dfrac{m}{k}\sqrt{\dfrac{\beta}{\theta}}k + \sqrt{1-\alpha}\dfrac{m}{k}\sqrt{1-\beta}k}$

$$= \dfrac{1}{\sqrt{\alpha\beta}+\sqrt{(1-\alpha)(1-\beta)}}$$

Fisher's Ideal index $= 1$

The Geometric Mean index $= \left(\sqrt{\dfrac{\beta}{\alpha}}\right)^{\alpha} \left(\sqrt{\dfrac{1-\beta}{1-\alpha}}\right)^{1-\alpha}$

Given the properties of the Cobb-Douglas utility function with given expenditure and the unchanged elliptical production possibility frontier, the second period prices will settle such that the Fisher's Ideal price index will be unity for all $\alpha$, $\beta$. The Laspeyres index will always be less than unity (given $0 < \alpha$, $\beta < 1$), except that it will equal unity in the trivial case when $\alpha = \beta$. Similarly, the Paasche index will always be more than unity.

The Geometric Mean index will always be below the Laspeyres index. To see this, set $\sqrt{\dfrac{\beta}{\alpha}} = A$ and $\sqrt{\dfrac{1-\beta}{1-\alpha}} = B$ . Then both indices will, for a given $\alpha$, be functions of $A$ and $B$. The equation for the Lasperyres' index represents a plane in the space, *L,A,B*. That for the Geometric Mean index represents a surface concave from below (towards the *A* and *B* axes). It will have the same slope as the Laspeyres plane when $A = B = 1$ (implying $\alpha = \beta$), when the two indices will also have the same value. Therefore, for all other values of *A*, *B* (i.e. for all values of $\beta$ given $\alpha$), the Geometric index lies below the Laspeyres index.

# Appendix 2

**Simulation study of elementary aggregation formulae in the presence of supply-side and demand-side shocks**

**reproduced from section 2.5 of J Winton, R O'Neill and D Elliott, *Elementary Aggregate Indices and Lower Level Substitution Bias*, published by the ONS as Annex A to CPAC(12)15, 30 April 2012[106]**

We performed a simulation using an economic model with a representative consumer and representative producer to evaluate the performance of some price indices under certain conditions of consumer substitution behaviour and for given demand and supply side shocks. We assumed that for a representative consumer in a two good world the utility function is:

$$U = \left( \sum_{i \in G} b_{it}^{1/\sigma} q_{it}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

Where $b_{it}$ is a taste parameter for the $i$th good in period $t$, $\sigma$ is the elasticity of substitution, $q_{it}$ represents the quantity of the $i$th good consumed in period $t$ and $G$ is the set of two goods $G$={A,B}. We also assume that the consumer has a budget $m$ to spend on the two goods.

We assume a production possibility frontier for the representative producer of:

$$\kappa^2 = \theta_t q_{1t}^2 + \lambda_t q_{2t}^2$$

Where $\vartheta, \lambda$ are parameters used to represent the relative costs of the two goods.

Setting the values of the parameters $\kappa, m, \sigma, \lambda_t, \theta_t, b_{it}$ for $i = A, B$ it is possible to evaluate the impact of shocks to supply and demand separately and contemporaneously. From well known economic theory, such as that used in Courtney (2011) it can be shown that the optimal quantities consumed in any given time period $t$ are:

$$q_{At} = \kappa \left[ \theta_t + \left\{ \lambda_t^{1/2} \left( \frac{b_{Bt}}{b_{At}} \right)^{1/(1+\sigma)} \left( \frac{\theta_t}{\lambda_t} \right)^{\sigma/(\sigma+1)} \right\}^2 \right]^{-1/2}$$

$$q_{Bt} = \kappa \left[ \lambda_t + \left\{ \theta_t^{1/2} \left( \frac{b_{At}}{b_{Bt}} \right)^{1/(1+\sigma)} \left( \frac{\lambda_t}{\theta_t} \right)^{\sigma/(\sigma+1)} \right\}^2 \right]^{-1/2}$$

Hence the quantities can be expressed as functions of the parameters chosen to define the model. The optimal prices can then be defined as:

$$p_{At} = \frac{m}{\left[ \frac{\lambda_t q_{Bt}^2}{\theta_t q_{At}^2} + q_{At} \right]}$$

$$p_{Bt} = \frac{m}{\left[ \frac{\theta_t q_{At}^2}{\lambda_t q_{Bt}^2} + q_{Bt} \right]}$$

which are determined by the parameter values chosen to define the model.

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[106] Downloadable at, http://www.ons.gov.uk/ons/guide-method/development-programmes/other-development-work/consumer-prices-advisory-committee/index.html

If we allow the taste or supply parameters to change through time, in effect allowing for demand and supply shocks respectively, we will see new optimal quantities and prices emerge. Our measure of inflation is the minimum amount a consumer would need under the new prices in order to obtain the same level of utility obtained in the first period. This would then give us the cost of living index defined in the economic theory of index numbers. We are aware that if we did adjust the consumer's income by the index constructed prices and quantities would change again however we are only concerned with measuring the theoretical level of inflation inherent in our model.

It is possible to answer the question of what is the minimum expenditure required under new prices to obtain the same level of utility experienced before the prices changed using a Lagrangian minimisation. We minimise the expenditure subject to the constraint that $U_t = U_{t-1}$. This approach results in the following optimal quantities:

$$q_{At}^* = \frac{U_{t-1}}{\Phi^{\sigma/(\sigma-1)}} \text{ where } \Phi = b_{At}^{1/\sigma} + b_{Bt}\left(\frac{p_{At}}{p_{Bt}}\right)^{\sigma-1}\left(\frac{1}{b_{At}}\right)^{(\sigma-1)/\sigma}$$

$$q_{Bt}^* = \frac{U_{t-1}}{\Theta^{\sigma/(\sigma-1)}} \text{ where } \Theta = b_{Bt}^{1/\sigma} + b_{At}\left(\frac{p_{Bt}}{p_{At}}\right)^{\sigma-1}\left(\frac{1}{b_{Bt}}\right)^{(\sigma-1)/\sigma}$$

Hence the true measure of the cost of living can be expressed as:

$$P_t^* = \frac{p_{At}q_{At}^* + p_{Bt}q_{Bt}^*}{m}$$

which can be calculated from the parameters used to define the model.

From this we are able to obtain prices and quantities for each of our goods in two periods; we are also able to obtain the measure of the true COLI ($P_t^*$) between these periods. We can then use the actual price and quantity information to estimate the indices proposed in the index numbers literature.

We performed simulations in this manner allowing for a range of values of $\sigma$ and shocks to the demand and supply functions. For each $\sigma$ = (0.1, 0.2, 0.3, … 4.0) we simulate 500 replications of random supply, demand and, supply and demand side shocks calculating the true cost of living index and evaluating which out of five indices are closest to the true cost of living. Charts are presented below showing the proportion of replications that each index closest to the true cost of living for given values of sigma.

Box 1: 500 random supply side shocks only to goods A and B in period 2

**Proportion where Index is closest to COLI**

**Range of inflation estimates by Index for all repitions and values of sigma**

**Price and quantity of good A and B in periods 1 and 2**
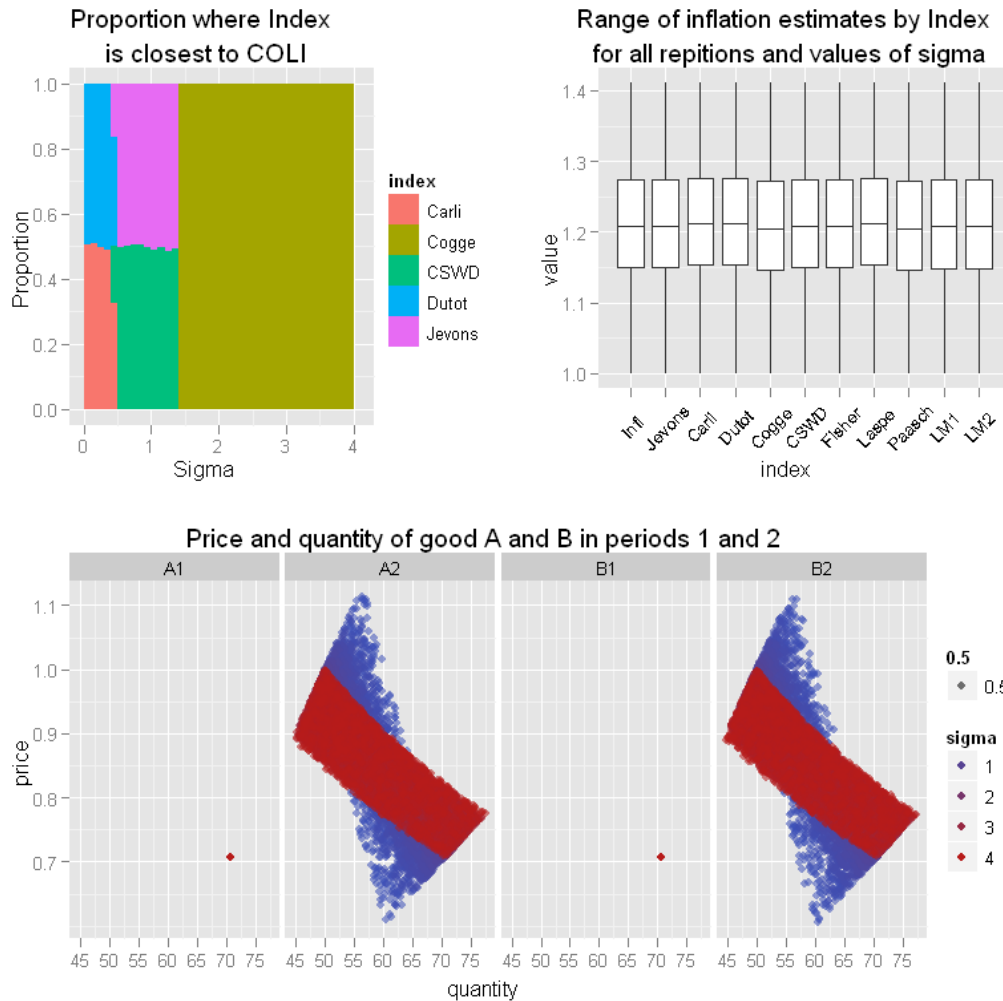
The range of inflation estimates for different indices over all replications and values of sigma is plotted for interest as are the price and quantity information in periods 1 and 2. The indices calculated for evaluation are Carli (average of price ratios), Cogge (short for Coggeshall, it is the harmonic mean of price ratios)[107], CSWD (Carruthers, Sellwood, Ward, Dalén or geometric mean of the Carli and Coggeshall), Dutot (ratio of average prices) and the Jevons (geometric mean of price ratios). The estimates for the Fisher, Laspeyres, Paasche and both sides of equation 2 (LM1 and LM2 in the charts) are also presented, but not evaluated.

As can be seen in Box 1 both the Carli and Dutot perform well where $\sigma$ is less than 0.5 whilst all except the Coggeshall perform well for $\sigma$=0.5 (although Carli and Dutot are more often closer than the CSWD and Jevons). For 0.5< $\sigma$ <1.5 CSWD and Jevons are closest to the true cost of living and the Coggeshall is closest where $\sigma$ ≥1.5. These results are as expected and discussed in section 2.2. Balk (2002) also notes that the harmonic mean is a form of the generalised mean with $\sigma$ =2.

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[107] This is most commonly referred to as the harmonic mean of price ratios, but we give it the name of Coggeshall just for presentation purposes.

Box 2: 500 random demand side shocks in period 2
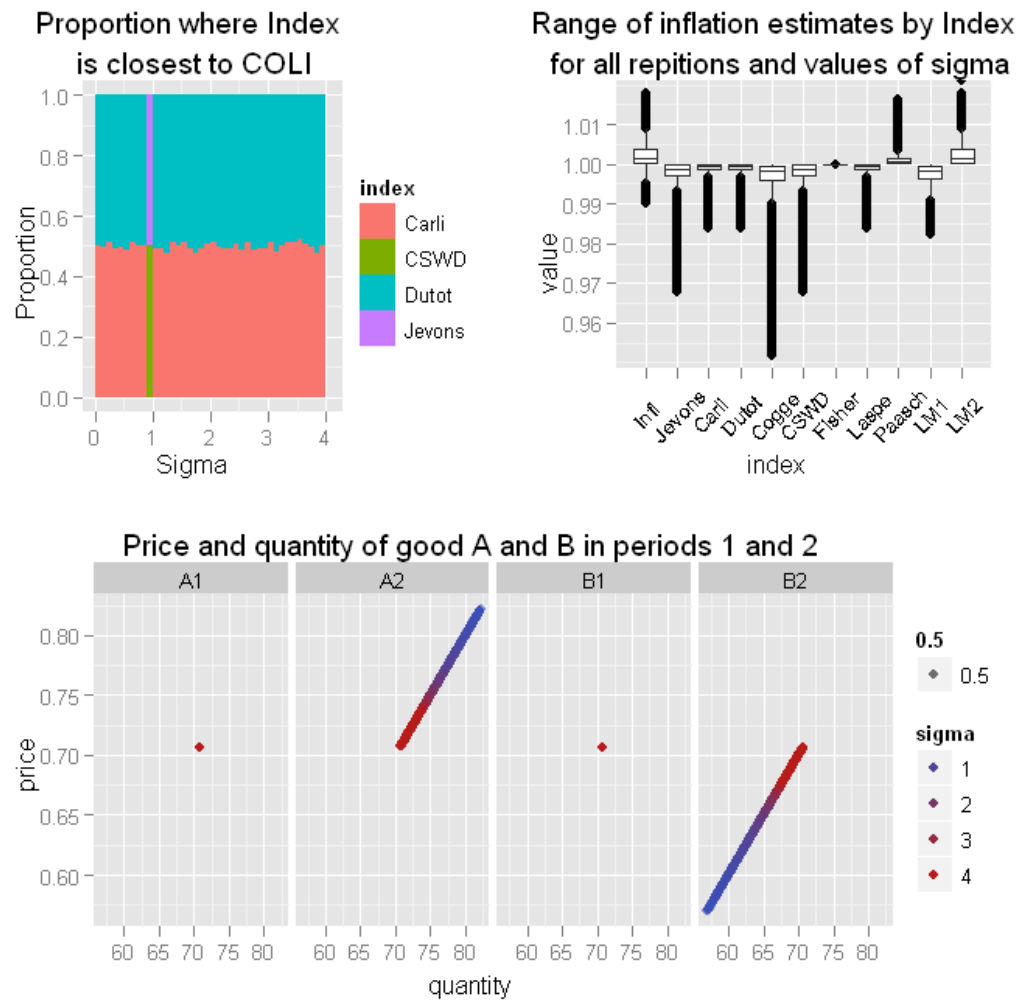
Box 2 shows the results for a demand side shock only.  As can be seen the Carli and Dutot perform well for all values of sigma except the case where $\sigma$ =1.  In the case of demand side effects only then all indices without quantity information underestimate inflation.  Box 3 shows the case for a demand and supply side shocks the magnitudes used are in the order of shocks used in Boxes 1 and 2. This particular combination of demand and supply side shocks shows that Carli and Dutot perform well for most values of sigma (dropping off slightly as the value of sigma rises). The results of this simulation are purely demonstrative and if various parameters and relative magnitudes of demand and supply side shocks are adjusted different indices can be shown to be better or worse.  The results in this section show the complications that arise in trying to appropriately evaluate indices for use as EAs using an economic approach.

Box 3: 500 random demand side shocks in period 2 and supply side shocks to good A and B in period 2.



Proportion where Index is closest to COLI

Range of inflation estimates by Index for all repitions and values of sigma

Price and quantity of good A and B in periods 1 and 2
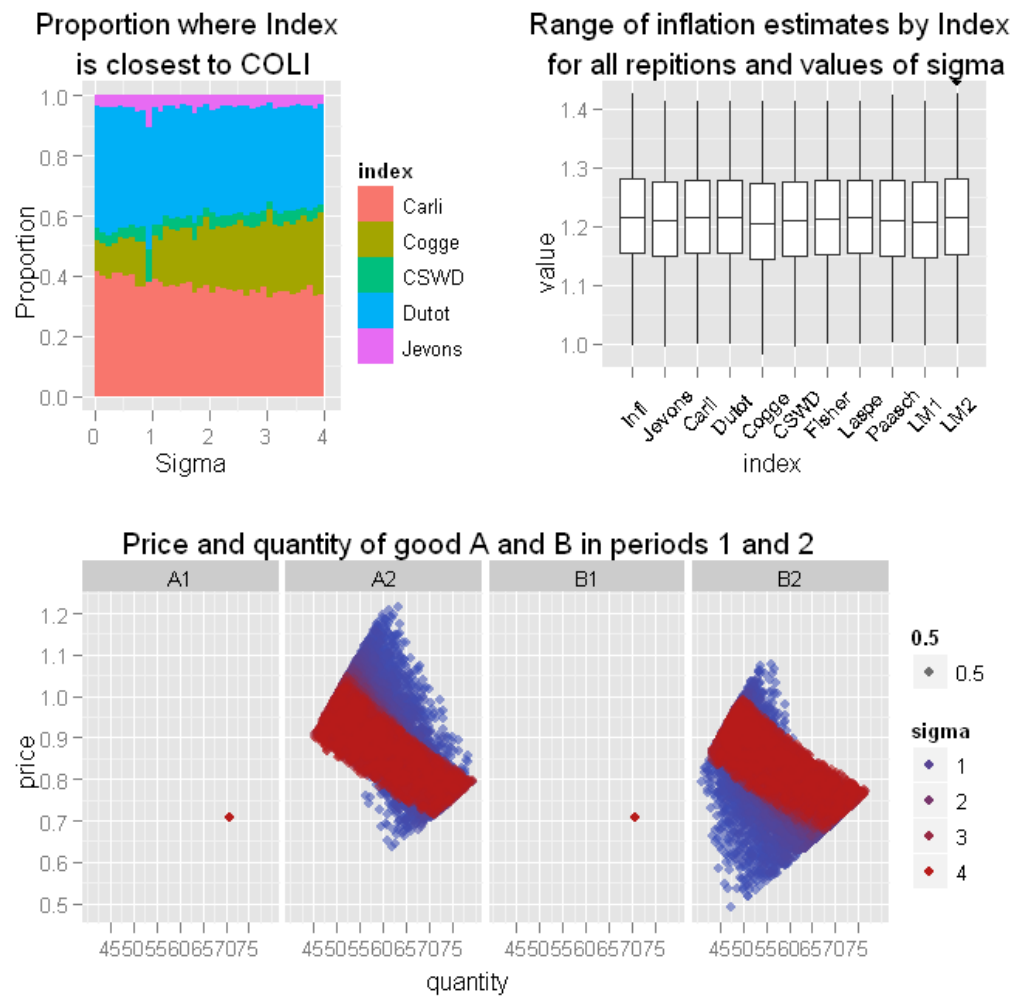
# Appendix 3

**Choice of target index in the Sampling Approach when the choice is restricted to an unweighted index:  Results from  Sections 3.2 and 3.3 of J Winton, R O'Neill and D Elliott, *Elementary Aggregate Indices and Lower Level Substitution Bias***

The ONS research team testing the Sampling Approach considered that the sampling results for the Fisher index were data-dependent, "because there is no reason to believe that the average increase in prices as measured by an index that includes weighting information based on expenditure shares should show the same as a simple average such as the Jevons or Carli." In that case, one would need to choose an unweighted index as the target index.  This makes their other research paper, *Elementary Aggregate Indices and Lower Level Substitution Bias*, in which they used the same alcohol data set, relevant.   Here, they were not using a sampling approach, but were calculating various aggregation indices using the whole data set.  For each aggregation index they therefore ended up with an index value for each period for each item. They then compared how closely they matched their target index, which they took to be the Fisher index.  In this case, they used three alternative measures of closeness (different loss functions), not only the MSE, but also the mean absolute deviation (MAD) and quasi-likelihood loss function (QLIKE).

Their results showing how close the Carli, Dutot and Jevons were to the Fisher index at the individual item level are in the table below:

**Table - Performance against Fisher Index**

| Index | Performance against Fisher (rank in Brackets) | | |
|---|---|---|---|
| | MSE | MAD | QLIKE × 1000 |
| Dutot | 11.33  ( 4) | 2.41  ( 4) | 0.51  ( 5) |
| Carli | 11.34  ( 5) | 2.44  ( 5) | 0.49  ( 4) |
| Jevons | 18.55  ( 8) | 2.89  ( 6) | 0.82  ( 8) |

By all measures, the Dutot and Carli perform similarly and are closer to the Fisher index than is the Jevons.

The ONS research team also looked at the results if the item indices were aggregated together to look at their performance against the Fisher for each class within the alcohol group and for the alcohol group as a whole.

**Table 4 - Performance of Higher Level**

| | Group (all alcohol level) | | | Average of Class levels | | |
|---|---|---|---|---|---|---|
| | Performance against Fisher (MAD) | | Correct 12 month Growth Directions | Performance against Fisher (MAD) | | Correct 12 month Growth Directions |
| | Index Level | 12 Month Growth | Out of a possible 22 | Index Level | 12 Month Growth | Out of a possible 22 |
| Carli | 0.54  (4) | 0.70  (4) | 18  (4) | 0.89  (4) | 1.02  (6) | 17.7  (5) |
| Dutot | 1.22  (5) | 0.75  (5) | 15  (7) | 1.19  (5) | 1.06  (7) | 18.0  (4) |
| Jevons | 1.66  (6) | 0.81  (6) | 17  (6) | 1.55  (6) | 1.17  (8) | 17.0  (8) |

At this upper index level, the Carli does better than the Dutot in approximating the Fisher index, while both continue to outperform the Jevons. Only when it comes to having a growth direction in the same direction as the Fisher do all three indices perform about equally well.

This implies that if one has qualms about picking an expenditure-weighted index as the target index, but wants a target population index that best approximates the Fisher index, then one should choose the Carli as one's population target index.

Returning now to the sampling study, the same alcohol data showed, unsurprisingly, that the sample Carli was a much less biased estimator of the population Carli than was the sample Jevons, with an average relative bias of -0.04% compared to -0.74%. However, when it came to their efficiency, there was little to choose between them, with the Jevons performing better for small samples (less than 9) and the Carli better for larger samples.

# Appendix 4

## Examples of Chain Drift with Oscillating Inflation and Smooth Inflation

As discussed in Section 10, the chain drift by which a chained price index differs from its direct counterpart can be either positive or negative. The chained index will lie above the direct index for oscillating inflation, and below it for relatively smooth inflation.

Papers which emphasise the potential for positive chain drift usually contain made-up examples with extreme price oscillation, such as that in Table A4.1 below, which is adapted from a table in the article by Roe and Fenwick of the ONS, published in January 2004 when the CPI became the new monetary target[108]

| Table A4.1: Oscillating Inflation | | | | |
|---|---|---|---|---|
| | price 1 | price 2 | arithmetic mean | geometric mean |
| Dec 2002 | 100 | 100 | 100 | 100 |
| Jan 2003 | 60 | 80 | | |
| *price relative* | *0.6* | *0.8* | *0.7* | *0.693* |
| Jan 2003 | 60 | 80 | | |
| Feb 2003 | 90 | 110 | 100 | 99.5 |
| *price relative* | *1.5* | *1.375* | *1.4375* | *1.4361* |

| Price Index | Carli, Direct | Carli, Chained | Jevons |
|---|---|---|---|
| Jan 2003 (Dec 2002=100) | 70 | | 69.3 |
| Feb 2003 (Jan 2003=100) | 143.75 | | 143.61 |
| Feb 2003 (Dec 2002=100) | 100 | $\frac{143.75 \times 70}{100} = 100.6$ | 99.5 |

With these prices, we first calculate the price index for January 2003, using December 2003 as our base month, and we do this using both the Carli formula and the Jevons formula. As always, the Jevons is below the Carli.

Next, we calculate the price index for February 2003, using January 2003 as our base month, using the same two alternative formulae.

Now we can calculate the price index for February 2003, with December 2002 as our base month. We can do this directly, in which case the Carli gives us an unchanged index of 100, and the Jevons a lower index of 99.5.

Or we can calculate it as a chained index, by multiplying together the two month-to-month indices. Algebraically, using the Jevons always gives the same result for the chained as for the direct index. But with the Carli: the chained index is 100.6, above the direct index. This,

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[108]D. Roe and D. Fenwick, The new inflation target: the statistical perspective, *Economic Trends*, Jan 2004.

according to critics of the RPI, shows the upward bias from the non-transitivity of the Carli. If we compare the chained indices, the Carli is 100.6, and the Jevons is 99.5, and fully one half of the formula-effect difference between them is accounted for by the non-transitivity of the Carli.

Notice that to get this result, our example has used some extreme price-bouncing: prices have declined from 100 to 60 or 80 and then bounced back straight back up again. That's pardonable, because we want an example with a clear-cut result, but it does mean that the effect will be much lower in practice.

Notice also that, as is usually the case, the example given uses month-to-month chaining. Doing so makes price-bouncing behaviour more plausible: perhaps January was a month with a lot of sales, so it's possible that prices were reduced and then bounced back up. But, in fact, most countries use annual (or even less frequent) chaining. It would be much more realistic, in this example, to make January 2002 rather than December 2002 our initial base month. Then, one would expect January 2003 to look rather similar to January 2002, so one would be much less likely to get price-bouncing behaviour.

But actually, there is no theoretical reason why the effect on chain drift should be positive rather than negative. Specifically, if inflation is relatively smooth, then one will get a negative effect on chain drift. By altering the numbers in the previous table we get:

| Table A4.2: Smooth Inflation | | | | |
|---|---|---|---|---|
| | price 1 | price 2 | arithmetic mean | geometric mean |
| Dec 2002 | 100 | 100 | 100 | 100 |
| Jan 2003 | 120 | 110 | | |
| *price relative* | *1.2* | *1.1* | *1.15* | *1.149* |
| Jan 2003 | 120 | 110 | | |
| Feb 2003 | 160 | 120 | 140 | 138.56 |
| *price relative* | *1.33* | *1.09* | *1.21* | *1.206* |

| Price Index | Carli, Direct | Carli, Chained | Jevons |
|---|---|---|---|
| Jan 2003 (Dec 2002=100) | 115 | | 114.89 |
| Feb 2003 (Jan 2003=100) | 121.21 | | 120.60 |
| Feb 2003 (Dec 2002=100) | 140 | $\frac{121.21 \times 115}{100} = 139.4$ | 138.6 |

Here we have relatively smooth, but not uniform, inflation. Again, we have had quite large price movements to get a clear-cut effect. With smooth inflation, chaining with the Carli formula results in negative chain drift. In this case, it halves the formula effect difference relative to the Jevons.

# Appendix 5

**Results from *Comparing Class level Chain Drift for Different Elementary Aggregate Formulae Using Locally Collected CPI Data*,
taken from *Chain Drift Synopsis*, RSS Statsusernet post by Gareth Jones, 6 February 2015**

| Class ID | Class Description | July/ May chained Jevons | July/ May chained Carli | July/ May chained Dutot | Drift: Jevons | Drift: Carli | Drift: Dutot | RPI Formula | year | CPI Weight |
|---|---|---|---|---|---|---|---|---|---|---|
| 10101 | BREAD & CEREALS | 100.05 | 101.40 | 99.92 | 0.15 | 1.07 | 0.09 | Dutot | 2009 | 17 |
| 10102 | MEAT | 99.80 | 100.86 | 99.82 | 0.10 | 0.52 | 0.08 | Dutot | 2009 | 23 |
| 10103 | FISH | 101.22 | 103.53 | 100.83 | 0.20 | 1.69 | 0.15 | Dutot | 2009 | 5 |
| 10104 | MILK, CHEESE & EGGS | 100.16 | 101.38 | 100.10 | 0.11 | 0.87 | 0.06 | Dutot | 2009 | 15 |
| 10105 | OILS & FATS | 99.41 | 100.05 | 99.37 | 0.05 | 0.17 | 0.04 | Dutot | 2009 | 2 |
| 10106 | FRUIT | 95.81 | 97.38 | 95.38 | 1.06 | 1.57 | 0.97 | Dutot | 2009 | 10 |
| 10107 | VEGETABLES INCLUDING POTATOES AND OTHER TUBERS | 97.31 | 98.71 | 97.33 | -0.01 | 0.63 | -0.02 | Dutot | 2009 | 16 |
| 10108 | SUGAR, JAM, HONEY, SYRUPS, CHOCOLATE AND CONFECTIONERY | 100.77 | 101.75 | 100.64 | 0.14 | 0.71 | 0.10 | Dutot | 2009 | 13 |
| 10109 | FOOD PRODUCTS | 101.38 | 103.34 | 101.21 | 0.14 | 1.22 | 0.11 | Dutot | 2009 | 3 |
| 10201 | COFFEE, TEA, COCOA | 102.16 | 104.07 | 101.30 | -0.01 | 0.99 | -0.01 | Dutot | 2009 | 4 |
| 10202 | MINERAL WATERS, SOFT DRINKS AND JUICES | 100.65 | 101.61 | 100.57 | 0.09 | 0.63 | 0.08 | Dutot | 2009 | 10 |
| 20101 | SPIRITS | 99.42 | 100.65 | 99.57 | 0.08 | 0.75 | 0.07 | Dutot | 2009 | 6 |
| 20102 | WINE (INC PERRY) | 98.58 | 99.78 | 98.71 | 0.06 | 0.55 | 0.07 | Dutot | 2009 | 10 |
| 20103 | BEER | 98.61 | 99.26 | 98.66 | 0.05 | 0.33 | 0.04 | Dutot | 2009 | 5 |
| 20200 | TOBACCO | 100.06 | 100.06 | 100.06 | 0.00 | 0.00 | 0.00 | Dutot | 2009 | 23 |
| 30102 | GARMENTS | 94.84 | 97.35 | 94.65 | -0.08 | 0.47 | -0.06 | Mostly Carli | 2009 | 44 |
| 30103 | OTHER ARTICLES OF CLOTHING & CLOTHING ACCESSORIES | 98.07 | 99.40 | 97.40 | 0.03 | 0.03 | 0.03 | Mixture | 2009 | 3 |
| 30104 | DRY_CLEANING, REPAIR AND HIRE OF CLOTHING | 100.71 | 100.80 | 100.76 | 0.00 | 0.07 | 0.00 | Dutot | 2009 | 1 |
| 30200 | FOOTWEAR INCLUDING REPAIRS | 97.75 | 99.34 | 97.66 | -0.03 | 0.23 | -0.03 | Carli | 2009 | 9 |
| 40100 | ACTUAL RENTS FOR HOUSING | 99.77 | 99.83 | 99.67 | 0.00 | 0.00 | 0.00 | Mixture | 2009 | 51 |
| 40301 | PRODUCTS FOR THE REGULAR MAINTENANCE AND REPAIR OF DWELLING | 100.85 | 101.12 | 100.80 | 0.00 | 0.02 | 0.00 | Mixture | 2009 | 10 |
| 40302 | SERVICES FOR THE REGULAR MAINTENANCE AND REPAIR OF DWELLING | 99.34 | 100.30 | 99.66 | 0.08 | 0.72 | 0.20 | Mixture | 2009 | 8 |
| 40503 | LIQUID FUELS | 100.47 | 100.75 | 100.40 | 0.03 | 0.15 | 0.03 | Carli | 2009 | 2 |
| 40504 | SOLID FUELS | 98.76 | 98.87 | 98.78 | 0.00 | 0.00 | 0.00 | Dutot | 2009 | 1 |
| 50101 | FURNITURE, FURNISHINGS | 97.72 | 101.55 | 96.54 | 0.17 | 2.72 | 0.32 | Carli | 2009 | 21 |
| 50200 | HOUSEHOLD TEXTILES | 97.61 | 98.70 | 96.22 | 0.00 | 0.25 | 0.00 | Carli | 2009 | 7 |

| 50301 | MAJOR HOUSEHOLD WHETHER ELECTRIC OR NOT AND SMALL ELECTRICAL APPLIANCES | 99.68 | 100.26 | 99.36 | 0.01 | 0.16 | 0.01 | Carli | 2009 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| 50303 | REPAIR OF HOUSEHOLD APPLIANCES | 101.27 | 101.77 | 101.57 | 0.04 | 0.21 | 0.03 | Mixture | 2009 | 1 |
| 50400 | GLASSWARE, TABLEWARE AND HOUSEHOLD UTENSILS | 97.64 | 98.52 | 96.55 | 0.00 | 0.10 | 0.00 | Mixture | 2009 | 5 |
| 50500 | TOOLS AND EQUIPMENT FOR HOUSE AND GARDEN | 99.72 | 100.03 | 99.36 | 0.00 | 0.03 | 0.00 | Mixture | 2009 | 6 |
| 50601 | NON-DURABLE HOUSEHOLD GOODS | 100.18 | 101.61 | 100.26 | 0.13 | 0.64 | 0.11 | Dutot | 2009 | 6 |
| 50602 | DOMESTIC SERVICES AND HOME CARE SERVICES | 99.86 | 100.20 | 99.78 | 0.02 | 0.20 | 0.03 | Carli | 2009 | 5 |
| 60101 | PHARMACEUTICAL PRODUCTS | 100.13 | 100.81 | 100.54 | 0.01 | 0.12 | 0.01 | Mainly Dutot | 2009 | 6 |
| 60102 | OTHER MEDICAL PRODUCTS THERAPEUTIC APPLIANCES AND EQUIPMENT | 99.92 | 100.07 | 99.83 | 0.01 | 0.01 | 0.01 | Mixture | 2009 | 4 |
| 60201 | MEDICAL SERVICES AND PARAMEDICAL SERVICES | 100.46 | 100.81 | 100.44 | 0.01 | 0.15 | 0.01 | Carli | 2009 | 2 |
| 60202 | DENTAL SERVICES | 101.00 | 101.30 | 100.75 | 0.01 | 0.00 | 0.00 | Carli | 2009 | 2 |
| 60300 | HOSPITAL SERVICES | 101.48 | 101.77 | 101.53 | 0.03 | 0.17 | 0.02 | Mixture | 2009 | 8 |
| 70102 | MOTOR CYCLES AND BICYCLES | 98.92 | 99.19 | 99.69 | -0.01 | -0.08 | 0.00 | Carli | 2009 | 3 |
| 70201 | SPARE PARTS & ACCESSORIES | 100.98 | 101.17 | 100.83 | 0.00 | 0.01 | 0.00 | Mostly Carli | 2009 | 5 |
| 70202 | FUELS & LUBRICANTS | 97.79 | 98.84 | 98.98 | -0.01 | 0.01 | -0.01 | Dutot | 2009 | 34 |
| 70203 | MAINTENANCE & REPAIRS | 100.43 | 100.53 | 100.45 | 0.00 | 0.00 | 0.00 | Mostly Carli | 2009 | 23 |
| 70204 | OTHER SERVICES IN RESPECT OF PERSONAL TRANSPORT EQUIPMENT | 100.48 | 100.88 | 100.36 | 0.00 | 0.09 | 0.00 | Mixture | 2009 | 7 |
| 70302 | PASSENGER TRANSPORT BY ROAD | 101.19 | 101.51 | 101.01 | 0.00 | 0.01 | 0.01 | Mixture | 2009 | 14 |
| 80200 | TELEPHONE AND TELEFAX EQUIPMENT AND SERVICES | 98.72 | 99.19 | 99.42 | 0.01 | 0.08 | 0.00 | Mixture | 2009 | 22 |
| 90101 | EQUIPMENT FOR THE RECEPTION & REPRODUCTION OF SOUND & PICTURE | 97.14 | 97.74 | 98.45 | -0.02 | 0.05 | 0.00 | Carli | 2009 | 6 |
| 90102 | PHOTOGRAPHIC AND CINEMATOGRAPHIC EQUIPMENT AND OPTICAL INSTRUMENTS | 99.41 | 99.49 | 99.62 | 0.00 | 0.00 | 0.00 | Mixture | 2009 | 4 |
| 90104 | RECORDING MEDIA | 98.40 | 99.97 | 99.13 | 0.00 | 0.17 | 0.01 | Mostly Carli | 2009 | 7 |
| 90105 | REPAIR OF AUDIO-VISUAL PHOTOGRAPHIC AND INFORMATION PROCESSING EQUIPMENT | 100.49 | 101.84 | 100.55 | 0.08 | 0.57 | 0.04 | Carli | 2009 | 1 |

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|---|---|---|---|---|---|---|---|---|---|
| 90201 | MAJOR DURABLES FOR INDOOR AND OUTDOOR RECREATION INCLUDING MUSICAL INSTRUMENTS | 105.14 | 125.60 | 108.15 | 3.61 | 19.21 | 7.27 | Mostly Carli | 2009 | 9 |
| 90301 | GAMES TOYS AND HOBBIES | 100.75 | 101.52 | 101.21 | 0.00 | 0.10 | 0.00 | Mixture | 2009 | 20 |
| 90302 | EQUIPMENT FOR SPORT CAMPING AND OPEN-AIR RECREATION | 101.03 | 102.13 | 100.88 | 0.00 | 0.20 | 0.00 | Mostly Carli | 2009 | 4 |
| 90303 | GARDEN PLANTS AND FLOWERS | 100.12 | 101.11 | 99.56 | -0.03 | 0.31 | -0.02 | Mostly Dutot | 2009 | 5 |
| 90304 | PETS AND RELATED PRODUCTS INCLUDING VETERINARY AND OTHER SERVICES FOR PETS | 99.50 | 99.70 | 99.66 | 0.00 | 0.10 | 0.00 | Mixture | 2009 | 8 |
| 90401 | RECREATIONAL AND SPORTING SERVICES | 100.09 | 100.53 | 100.48 | 0.03 | 0.02 | 0.02 | Mixture | 2009 | 11 |
| 90402 | CULTURAL SERVICES | 100.30 | 104.07 | 98.50 | -0.17 | 0.86 | -0.21 | Mixture | 2009 | 21 |
| 90503 | MISC. PRINTED MATTER AND STATIONERY AND DRAWING MATERIALS | 99.77 | 100.13 | 99.89 | 0.00 | 0.10 | 0.00 | Mostly Dutot | 2009 | 6 |
| 110101 | RESTAURANTS & CAFES | 100.21 | 100.30 | 100.22 | 0.00 | 0.02 | 0.00 | Mostly Dutot | 2009 | 100 |
| 110102 | CANTEENS | 100.47 | 100.62 | 100.56 | 0.00 | 0.00 | 0.00 | Mixture | 2009 | 11 |
| 110200 | ACCOMMODATION SERVICES | 102.37 | 105.38 | 101.55 | 0.09 | 1.85 | 0.04 | Mixture | 2009 | 17 |
| 120101 | HAIRDRESSING SALONS & PERSONAL GROOMING ESTABLISHMENTS | 99.93 | 100.06 | 99.88 | 0.00 | 0.01 | 0.00 | Carli | 2009 | 8 |
| 120102 | APPLIANCES, ARTICLES & PRODUCTS FOR PERSONAL CARE | 100.73 | 102.17 | 100.86 | 0.04 | 0.59 | 0.03 | Mixture | 2009 | 23 |
| 120301 | JEWELLERY CLOCKS AND WATCHES | 99.46 | 100.00 | 99.65 | 0.00 | 0.08 | 0.00 | Carli | 2009 | 8 |
| 120302 | OTHER PERSONAL EFFECTS | 97.74 | 98.78 | 98.57 | -0.01 | 0.21 | 0.00 | Carli | 2009 | 3 |
| 120400 | SOCIAL PROTECTION | 101.34 | 102.01 | 101.28 | 0.03 | 0.32 | 0.02 | Mixture | 2009 | 11 |
| 120700 | OTHER SERVICES NEC | 100.63 | 100.99 | 100.51 | 0.02 | 0.27 | 0.03 | Mixture | 2009 | 11 |
| 10101 | BREAD & CEREALS | 100.56 | 101.73 | 100.44 | 0.10 | 0.65 | 0.07 | Dutot | 2010 | 16 |
| 10102 | MEAT | 101.45 | 102.90 | 101.27 | 0.15 | 0.80 | 0.13 | Dutot | 2010 | 22 |
| 10103 | FISH | 103.85 | 105.81 | 103.68 | 0.12 | 1.19 | 0.01 | Dutot | 2010 | 4 |
| 10104 | MILK, CHEESE & EGGS | 100.68 | 101.78 | 100.70 | 0.07 | 0.73 | 0.05 | Dutot | 2010 | 14 |
| 10105 | OILS & FATS | 106.66 | 109.19 | 106.41 | 0.05 | 0.88 | 0.04 | Dutot | 2010 | 2 |
| 10106 | FRUIT | 100.15 | 102.78 | 99.54 | 0.50 | 0.96 | 0.47 | Dutot | 2010 | 9 |
| 10107 | VEGETABLES INCLUDING POTATOES AND OTHER TUBERS | 102.41 | 104.27 | 102.27 | 0.36 | 1.16 | 0.33 | Dutot | 2010 | 15 |
| 10108 | SUGAR, JAM, HONEY, SYRUPS, CHOCOLATE AND CONFECTIONERY | 99.65 | 100.43 | 99.71 | 0.04 | 0.32 | 0.03 | Dutot | 2010 | 11 |
| 10109 | FOOD PRODUCTS | 105.40 | 108.14 | 104.59 | -0.07 | 1.68 | -0.06 | Dutot | 2010 | 3 |
| 10201 | COFFEE, TEA, COCOA | 99.36 | 100.46 | 99.45 | 0.07 | 0.82 | 0.07 | Dutot | 2010 | 3 |
| 10202 | MINERAL WATERS, SOFT DRINKS AND JUICES | 101.84 | 103.99 | 101.89 | 0.25 | 1.69 | 0.19 | Dutot | 2010 | 9 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 20101 | SPIRITS | 99.32 | 100.70 | 99.28 | -0.01 | 0.42 | -0.01 | Dutot | 2010 | 5 |
| 20102 | WINE (INC PERRY) | 100.68 | 102.75 | 100.44 | 0.12 | 1.03 | 0.11 | Dutot | 2010 | 9 |
| 20103 | BEER | 99.25 | 99.71 | 99.29 | 0.01 | 0.12 | 0.01 | Dutot | 2010 | 4 |
| 20200 | TOBACCO | 99.59 | 99.60 | 99.59 | -0.01 | 0.00 | -0.01 | Dutot | 2010 | 22 |
| 30102 | GARMENTS | 92.04 | 96.56 | 92.18 | -0.11 | 1.04 | -0.07 | Mostly Carli | 2010 | 43 |
| 30103 | OTHER ARTICLES OF CLOTHING & CLOTHING ACCESSORIES | 94.03 | 96.84 | 94.55 | -0.06 | 0.42 | -0.04 | Mixture | 2010 | 3 |
| 30104 | DRY_CLEANING, REPAIR AND HIRE OF CLOTHING | 100.02 | 100.10 | 100.09 | 0.00 | 0.00 | 0.00 | Dutot | 2010 | 1 |
| 30200 | FOOTWEAR INCLUDING REPAIRS | 96.34 | 98.06 | 96.52 | 0.01 | 0.42 | 0.02 | Carli | 2010 | 9 |
| 40100 | ACTUAL RENTS FOR HOUSING | 99.97 | 100.01 | 99.98 | 0.00 | 0.00 | 0.00 | Mixture | 2010 | 54 |
| 40301 | PRODUCTS FOR THE REGULAR MAINTENANCE AND REPAIR OF DWELLING | 101.55 | 101.85 | 101.26 | 0.01 | 0.03 | 0.01 | Mixture | 2010 | 10 |
| 40302 | SERVICES FOR THE REGULAR MAINTENANCE AND REPAIR OF DWELLING | 100.27 | 100.37 | 100.16 | 0.00 | 0.00 | 0.00 | Mixture | 2010 | 7 |
| 40503 | LIQUID FUELS | 94.55 | 94.63 | 94.62 | 0.00 | 0.01 | 0.00 | Carli | 2010 | 2 |
| 40504 | SOLID FUELS | 98.32 | 98.42 | 98.34 | 0.00 | 0.02 | 0.00 | Dutot | 2010 | 1 |
| 50101 | FURNITURE, FURNISHINGS | 97.94 | 100.48 | 97.21 | 0.11 | 1.84 | 0.27 | Carli | 2010 | 19 |
| 50200 | HOUSEHOLD TEXTILES | 96.37 | 97.58 | 95.19 | 0.01 | 0.43 | 0.00 | Carli | 2010 | 8 |
| 50301 | MAJOR HOUSEHOLD WHETHER ELECTRIC OR NOT AND SMALL ELECTRICAL APPLIANCES | 100.62 | 101.70 | 100.39 | 0.01 | 0.61 | 0.01 | Carli | 2010 | 8 |
| 50303 | REPAIR OF HOUSEHOLD APPLIANCES | 100.34 | 100.40 | 100.33 | -0.01 | 0.00 | -0.01 | Mixture | 2010 | 1 |
| 50400 | GLASSWARE, TABLEWARE AND HOUSEHOLD UTENSILS | 97.86 | 98.77 | 96.62 | -0.01 | 0.14 | 0.00 | Mixture | 2010 | 6 |
| 50500 | TOOLS AND EQUIPMENT FOR HOUSE AND GARDEN | 100.69 | 101.55 | 100.86 | 0.02 | 0.08 | 0.02 | Mixture | 2010 | 7 |
| 50601 | NON-DURABLE HOUSEHOLD GOODS | 103.77 | 106.20 | 103.73 | 0.25 | 1.28 | 0.25 | Dutot | 2010 | 5 |
| 50602 | DOMESTIC SERVICES AND HOME CARE SERVICES | 100.19 | 100.20 | 100.16 | 0.00 | 0.00 | 0.00 | Carli | 2010 | 4 |
| 60101 | PHARMACEUTICAL PRODUCTS | 100.20 | 101.11 | 100.01 | 0.04 | 0.15 | 0.03 | Mainly Dutot | 2010 | 6 |
| 60102 | OTHER MEDICAL PRODUCTS THERAPEUTIC APPLIANCES AND EQUIPMENT | 99.72 | 100.09 | 99.54 | 0.01 | 0.14 | 0.01 | Mixture | 2010 | 4 |
| 60201 | MEDICAL SERVICES AND PARAMEDICAL SERVICES | 100.52 | 100.55 | 100.52 | 0.00 | 0.00 | 0.00 | Carli | 2010 | 2 |
| 60202 | DENTAL SERVICES | 100.43 | 100.47 | 100.34 | 0.00 | 0.00 | 0.00 | Carli | 2010 | 2 |
| 60300 | HOSPITAL SERVICES | 100.38 | 100.40 | 100.36 | 0.00 | 0.00 | 0.00 | Mixture | 2010 | 8 |
| 70102 | MOTOR CYCLES AND BICYCLES | 101.63 | 101.94 | 101.67 | 0.04 | 0.02 | 0.03 | Carli | 2010 | 3 |
| 70201 | SPARE PARTS & ACCESSORIES | 100.24 | 100.51 | 100.30 | 0.00 | 0.08 | 0.00 | Mostly Carli | 2010 | 5 |

| Code | Description | | | | | | | Method | Year | Count |
|---|---|---|---|---|---|---|---|---|---|---|
| 70202 | FUELS & LUBRICANTS | 98.78 | 99.96 | 98.92 | 0.03 | -0.01 | 0.02 | Dutot | 2010 | 41 |
| 70203 | MAINTENANCE & REPAIRS | 100.14 | 100.26 | 100.16 | 0.00 | 0.00 | 0.00 | Mostly Carli | 2010 | 25 |
| 70204 | OTHER SERVICES IN RESPECT OF PERSONAL TRANSPORT EQUIPMENT | 100.71 | 101.02 | 100.52 | 0.00 | 0.00 | 0.00 | Mixture | 2010 | 8 |
| 70302 | PASSENGER TRANSPORT BY ROAD | 101.24 | 101.35 | 100.98 | 0.00 | 0.00 | 0.00 | Mixture | 2010 | 14 |
| 80200 | TELEPHONE AND TELEFAX EQUIPMENT AND SERVICES | 97.78 | 103.28 | 95.46 | 0.04 | 5.17 | 0.01 | Mixture | 2010 | 24 |
| 90101 | EQUIPMENT FOR THE RECEPTION & REPRODUCTION OF SOUND & PICTURE | 98.23 | 98.93 | 99.50 | 0.01 | 0.05 | 0.01 | Carli | 2010 | 7 |
| 90102 | PHOTOGRAPHIC AND CINEMATOGRAPHIC EQUIPMENT AND OPTICAL INSTRUMENTS | | | | | | | Mixture | 2010 | 3 |
| | | | | | | | **no info.** | | | |
| 90104 | RECORDING MEDIA | 96.37 | 99.23 | 96.39 | 0.07 | 1.36 | 0.06 | Mostly Carli | 2010 | 6 |
| 90105 | REPAIR OF AUDIO-VISUAL PHOTOGRAPHIC AND INFORMATION PROCESSING EQUIPMENT | 99.73 | 99.75 | 99.58 | 0.00 | 0.00 | 0.00 | Carli | 2010 | 1 |
| 90201 | MAJOR DURABLES FOR INDOOR AND OUTDOOR RECREATION INCLUDING MUSICAL INSTRUMENTS | 99.99 | 100.10 | 100.07 | 0.00 | 0.00 | 0.00 | Mostly Carli | 2010 | 9 |
| 90301 | GAMES TOYS AND HOBBIES | 99.67 | 100.43 | 100.10 | 0.01 | 0.04 | 0.01 | Mixture | 2010 | 24 |
| 90302 | EQUIPMENT FOR SPORT CAMPING AND OPEN-AIR RECREATION | 97.81 | 99.56 | 97.44 | -0.07 | 0.11 | -0.06 | Mostly Carli | 2010 | 4 |
| 90303 | GARDEN PLANTS AND FLOWERS | 98.09 | 99.08 | 98.57 | -0.02 | 0.09 | -0.02 | Mostly Dutot | 2010 | 5 |
| 90304 | PETS AND RELATED PRODUCTS INCLUDING VETERINARY AND OTHER SERVICES FOR PETS | 100.52 | 100.70 | 100.49 | 0.01 | 0.10 | 0.02 | Mixture | 2010 | 7 |
| 90401 | RECREATIONAL AND SPORTING SERVICES | 100.58 | 100.60 | 100.85 | 0.00 | 0.00 | 0.00 | Mixture | 2010 | 10 |
| 90402 | CULTURAL SERVICES | 96.24 | 98.83 | 95.15 | 0.26 | 1.50 | 0.46 | Mixture | 2010 | 21 |
| 90503 | MISC. PRINTED MATTER AND STATIONERY AND DRAWING MATERIALS | 99.95 | 100.46 | 99.84 | 0.00 | 0.04 | 0.00 | Mostly Dutot | 2010 | 7 |
| 110101 | RESTAURANTS & CAFES | 100.22 | 100.34 | 100.26 | 0.00 | 0.02 | 0.00 | Mostly Dutot | 2010 | 98 |
| 110102 | CANTEENS | 100.50 | 100.66 | 100.66 | 0.00 | 0.02 | 0.00 | Mixture | 2010 | 11 |
| 110200 | ACCOMMODATION SERVICES | 104.29 | 105.44 | 104.70 | 0.76 | 1.42 | 0.37 | Mixture | 2010 | 17 |
| 120101 | HAIRDRESSING SALONS & PERSONAL GROOMING ESTABLISHMENTS | 100.47 | 100.53 | 100.51 | 0.00 | 0.01 | 0.00 | Carli | 2010 | 7 |

| 120102 | APPLIANCES, ARTICLES & PRODUCTS FOR PERSONAL CARE | 100.02 | 101.83 | 99.89 | 0.03 | 0.64 | 0.02 | Mixture | 2010 | 22 |
| 120301 | JEWELLERY CLOCKS AND WATCHES | 101.40 | 102.78 | 102.08 | -0.03 | 0.16 | -0.01 | Carli | 2010 | 7 |
| 120302 | OTHER PERSONAL EFFECTS | 95.54 | 97.08 | 94.93 | 0.02 | 0.47 | 0.03 | Carli | 2010 | 3 |
| 120400 | SOCIAL PROTECTION | 100.58 | 100.60 | 100.58 | 0.00 | 0.00 | 0.00 | Mixture | 2010 | 11 |
| 120700 | OTHER SERVICES NEC | 100.17 | 100.26 | 100.12 | 0.01 | 0.00 | 0.01 | Mixture | 2010 | 11 |