ADVISORY PANEL ON CONSUMER PRICES – TECHNICAL

Reply to paper APCP-T(19)01:
The use of unweighted indices in ONS's consumer price statistics

Purpose

1. This paper tries to shed light on the reasons behind the increase in the formula effect following the changes to collection methods for clothing in 2010, using published CPI microdata.

Actions

2. Members of the Panel are invited to:
   a) comment on whether the high variance in price relatives in January (month 13) is due to dispersion of prices over time or the replacement of products over the course of the 13 months
   b) advise on the collection methods for clothing and the use of the Carli index

Discussion

3. At its meeting on 11 January 2019, the Advisory Panels on Consumer Prices – Technical had a discussion on the use of unweighted indices in ONS's consumer price statistics. The underlying paper summarised the arguments about the relative merits and de-merits of the Carli, Jevons and Dutot indices.
4. The mathematical properties of the elementary aggregate formulae and their numerical relationships are reviewed in Chapter 8 of the HICP Methodological Manual (Balk/Mehrhoff, 2018). In what follows, the equation numbering refers to that publication.

   Mathematical properties

5. The **<u>Dutot index</u>** has the drawback of tending primarily to reflect the price development of products at relatively high prices.

$$P_D^{0t,mt} = \frac{\frac{1}{K}\sum_{k=1}^{K} p_k^{mt}}{\frac{1}{K}\sum_{k=1}^{K} p_k^{0t}} = \sum_{k=1}^{K} \frac{p_k^{mt}}{p_k^{0t}} \cdot \frac{p_k^{0t}}{\sum_{l=1}^{K} p_l^{0t}} \qquad (8.10a)$$

6. It appears that the Dutot index can be written as a weighted arithmetic mean of individual price relatives, its weights being relative prices in the price reference period. In the Dutot index, products with higher relative prices get a higher weight and products with lower relative prices get a lower weight. Thus, it is advisable to use the Dutot index only for elementary aggregates in which the relative prices exhibit small variance, i.e. the price levels are similar.
7. The **commensurability test** means that if the units of measurement for each product are changed, then the elementary aggregate index remains unchanged. The Dutot index fails this test, since the price levels are affected by the measurement unit. If there are heterogeneous products in the elementary aggregate, this is a rather serious failing and price statisticians should be careful when using this index under these conditions.
8. A quick glance at the formula of the **<u>Jevons index</u>** tells us that it is not a linear index. Nevertheless, the unique linear approximation to the Jevons index first described by Mehrhoff (2007), and independently devised by Balk (2008) as the

unweighted Walsh index, yields what is referred to as the Balk-Mehrhoff-Walsh (BMW) index. It is weighted by the square root of the inverse price relatives.

$$P_J^{0t,mt} = \left(\prod_{k=1}^{K} \frac{p_k^{mt}}{p_k^{0t}}\right)^{\frac{1}{K}} \approx \sum_{k=1}^{K} \frac{p_k^{mt}}{p_k^{0t}} \cdot \frac{\sqrt{p_k^{0t}/p_k^{mt}}}{\sum_{l=1}^{K} \sqrt{p_l^{0t}/p_l^{mt}}} = P_{BMW}^{0t,mt} \tag{8.11a}$$

9. Thus, this index is more robust with respect to the variance of relative prices within an elementary aggregate.
10. The **test of determinateness as to prices** means that if any single price tends to zero, then the index should not tend to zero or plus infinity. It can be verified that the Jevons index does not satisfy this test. Thus, when using the Jevons index, care must be taken to bound the prices away from zero in order to avoid a meaningless index number value.
11. The chained **Carli index** does not reduce to a direct Carli index.

$$CP_C^{0t,mt} = P_C^{0t,1t} \cdot P_C^{1t,2t} \cdot \ldots \cdot P_C^{(m-1)t,mt}$$
$$= \left(\frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{1t}}{p_k^{0t}}\right) \cdot \left(\frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{2t}}{p_k^{1t}}\right) \cdot \ldots \cdot \left(\frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{mt}}{p_k^{(m-1)t}}\right) \neq \frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{mt}}{p_k^{0t}} = P_C^{0t,mt} \tag{8.14}$$

12. The underlying phenomenon is that it does not satisfy the **circularity (transitivity) test**. This failing implies an indefinite bias, which in practice may show up as chain drift.

Numerical relationships

13. By expanding the Jevons index by a second-order Taylor series approximation around the arithmetic mean prices $p_k^{0t} = \bar{p}^{0t}$ and $p_k^{mt} = \bar{p}^{mt}$ for all $k = 1, \ldots, K$ (K being sufficiently large), one can verify that the **difference between the Dutot and Jevons indices** depends on the change over time of the squared coefficient of variation of individual prices.

$$P_J^{0t,mt} \approx P_D^{0t,mt}\left(1 + \frac{1}{2}\frac{\mathrm{Var}[p_k^{0t}]}{(\mathrm{E}[p_k^{0t}])^2} - \frac{1}{2}\frac{\mathrm{Var}[p_k^{mt}]}{(\mathrm{E}[p_k^{mt}])^2}\right)$$
$$\Leftrightarrow P_D^{0t,mt} \approx P_J^{0t,mt}\left(1 + \frac{1}{2}\frac{\mathrm{Var}[p_k^{0t}]}{(\mathrm{E}[p_k^{0t}])^2} - \frac{1}{2}\frac{\mathrm{Var}[p_k^{mt}]}{(\mathrm{E}[p_k^{mt}])^2}\right)^{-1} \tag{8.15}$$

14. Likewise, by expanding the Dutot index around the geometric mean prices $\ln p_k^{0t} = \ln \bar{p}^{0t}$ and $\ln p_k^{mt} = \ln \bar{p}^{mt}$, one obtains the following second-order approximate relationship.

$$P_D^{0t,mt} \approx P_J^{0t,mt}\left(1 - \frac{1}{2}\mathrm{Var}[\ln p_k^{0t}] + \frac{1}{2}\mathrm{Var}[\ln p_k^{mt}]\right) \tag{8.16}$$

15. However, whether the difference between the Dutot and Jevons indices is positive or negative, large or small, is an empirical matter. Still, Silver and Heravi (2007, J. Econometrics) show that this **difference depends on the change over time in price dispersion**. Some of the price dispersion will be due to product heterogeneity.
16. Define the k-th price relative as $r_k = p_k^{mt}/p_k^{0t}$. The second-order Taylor series approximation to the **difference between the Carli and Jevons indices** depends on the squared coefficient of variation of price relatives (see Chapter 20 of the CPI Manual; Diewert, 2004).

$$P_J^{0t,mt} \approx P_C^{0t,mt}\left(1 - \frac{1}{2}\frac{\mathrm{Var}[r_k]}{(\mathrm{E}[r_k])^2}\right)$$
$$\Leftrightarrow P_C^{0t,mt} \approx P_J^{0t,mt}\left(1 - \frac{1}{2}\frac{\mathrm{Var}[r_k]}{(\mathrm{E}[r_k])^2}\right)^{-1}$$

CPI Manual (20.27)

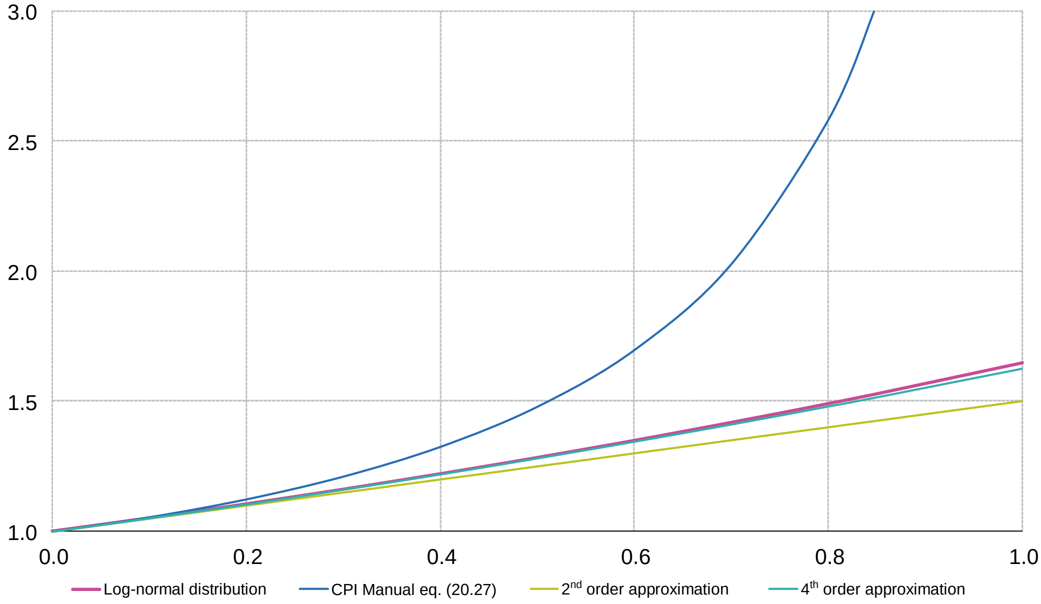
17. Again, expanding the Carli index around the geometric mean, one obtains the following new relationships, where σ^2 is the variance, γ_1 is the skewness and β_2 is the kurtosis (not excess) of the natural logarithms of price relatives.

$$P_C^{0t,mt} \approx P_J^{0t,mt}\left(1+\frac{\sigma^2}{2}+\frac{\gamma_1\,\sigma^3}{6}+\frac{\beta_2\,\sigma^4}{24}\right) \qquad \text{fourth–order}$$

$$P_C^{0t,mt} = P_J^{0t,mt}\exp\frac{\sigma^2}{2} \qquad \text{log-normal}$$

18. To the second order, the Carli index will exceed the Jevons index by half the variance of the natural logarithms of price relatives. This more straightforward transformation of price relatives has the advantage that the **typical skewness of the distribution is greatly reduced, making even the second-order approximation more accurate**.
19. Assume that price relatives are exactly log-normally distributed. The ratio between the Carli and Jevons indices then depends exponentially on the variance of the natural logarithms of price relatives. The above approximations work best around the geometric mean rather than the arithmetic mean of price relatives.

Ratio between the Carli and Jevons indices
as a function of the variance of the natural logarithms of price relatives



20. **Annex A** uses these new relationships to derive **the driving force behind chain drift in clothing**, referring to earlier work of the Panel (de Vincent-Humphreys, APCP-T(17)02). It further makes an attempt to link the variance in price relatives to **the proportion of within-year (13 months) replacements**. Finally yet importantly, it reflects upon the changes to collection methods for clothing, in particular **the replacement of products as 'comparable'**.
21. **Annex B** investigates the **difference between the direct Carli and chained Carli indices** and how this **relates to sales prices**.

Jens Mehrhoff
February, 2019

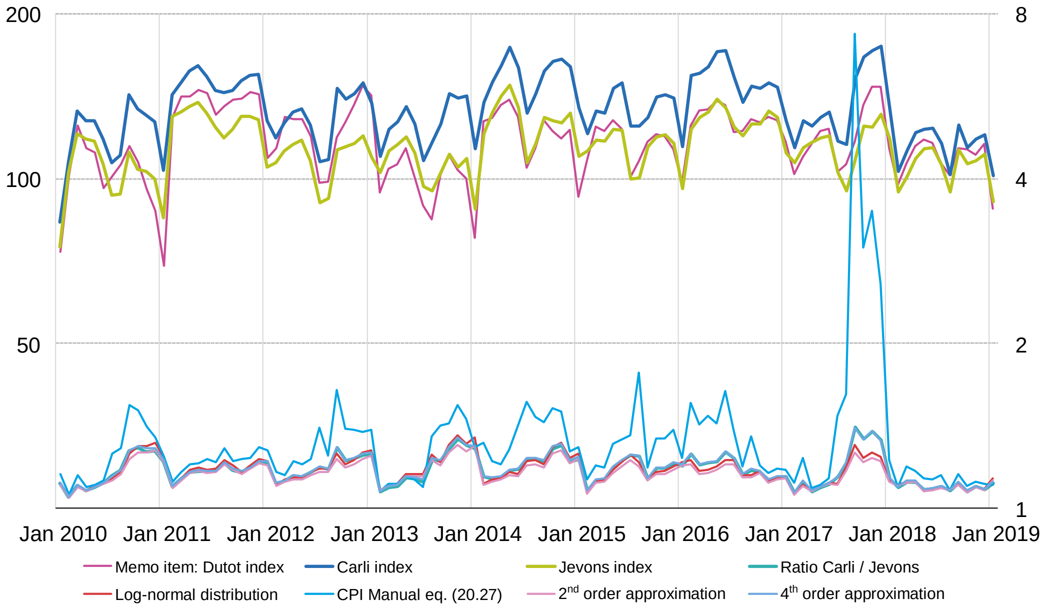
Disclaimer: This paper represents the author's personal opinions and does not necessarily reflect the views of the European Commission or its staff.

List of Annexes

Annex A: On the difference between the chained Carli and chained Jevons indices

1. Using published CPI microdata for women's vests/strappy tops from independent stores over the period from January 2010 to January 2019, direct as well as chained Carli and Jevons indices are calculated. Both RPI and CPI series are produced from the same set of price data; while RPI uses the arithmetic mean (Carli formula), CPI uses the geometric mean (Jevons formula).
2. Starting with the direct indices, the approximations set out in this paper are employed to the **ratio between Carli and Jevons**.

Price indices for women's vests/strappy tops (Jan t–1 = 100, log scale)
and ratio between the Carli and Jevons indices (right axis, log scale)



3. The long-term development in the RPI is driven by the direct indices in January of each year.

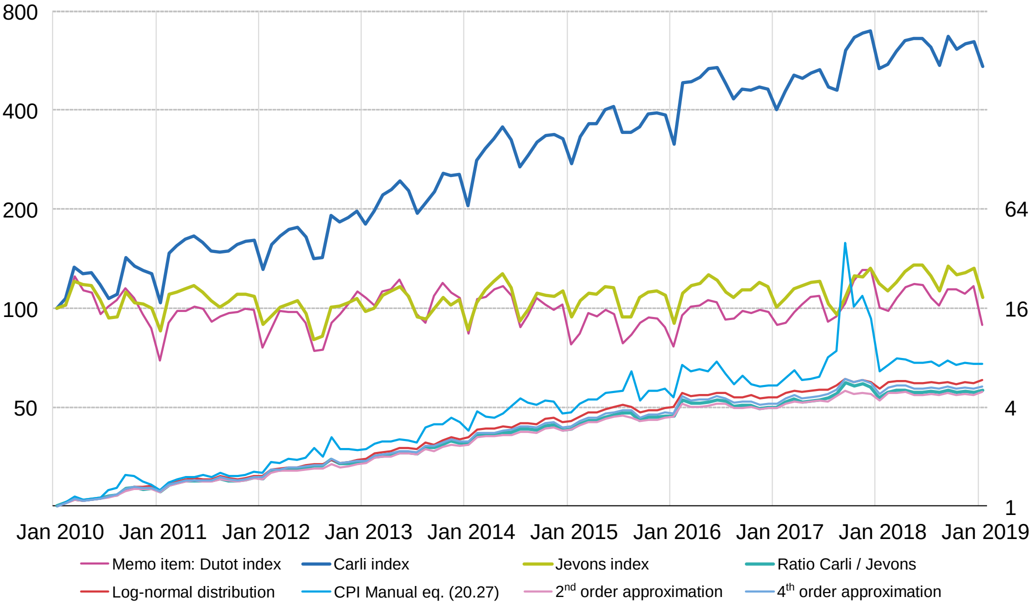
$$CP_C^{\text{Jan 2010},mt} = 100 \cdot P_C^{\text{Jan 2010,Jan 2011}} \cdot P_C^{\text{Jan 2011,Jan 2012}} \cdot \ldots \cdot P_C^{\text{Jan }t,mt}$$
$$= CP_J^{\text{Jan 2010},mt} \cdot \left(\frac{P_C^{\text{Jan 2010,Jan 2011}}}{P_J^{\text{Jan 2010,Jan 2011}}} \cdot \frac{P_C^{\text{Jan 2011,Jan 2012}}}{P_J^{\text{Jan 2011,Jan 2012}}} \cdot \ldots \cdot \frac{P_C^{\text{Jan }t,mt}}{P_J^{\text{Jan }t,mt}} \right)$$

4. The **ratios between the Carli and Jevons indices** can be approximated using any of the above relationships, e.g. to the second order.

$$\frac{CP_C^{\text{Jan 2010},mt}}{CP_J^{\text{Jan 2010},mt}} \approx \left(1 + \frac{\sigma^2 |^{\text{Jan 2010,Jan 2011}}}{2}\right) \cdot \left(1 + \frac{\sigma^2 |^{\text{Jan 2011,Jan 2012}}}{2}\right) \cdot \ldots \cdot \left(1 + \frac{\sigma^2 |^{\text{Jan }t,mt}}{2}\right)$$

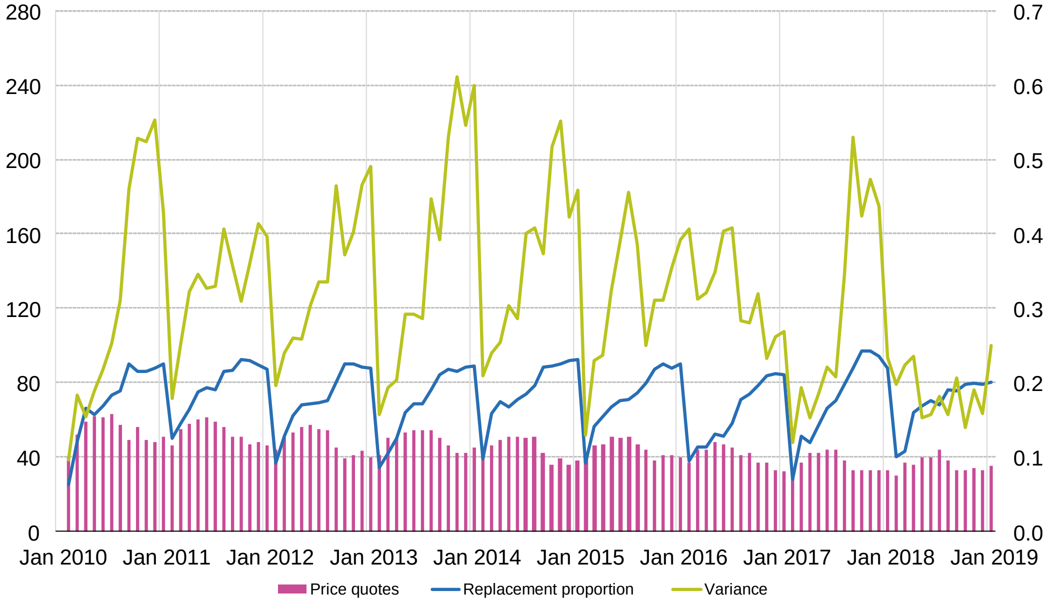
5. Hence, the difference between the direct Carli and direct Jevons indices in each January can be translated into the difference of the respective chained indices.

Price indices for women's vests/strappy tops (Jan 2010 = 100, log scale)
and ratio between the Carli and Jevons indices (right axis, log scale)



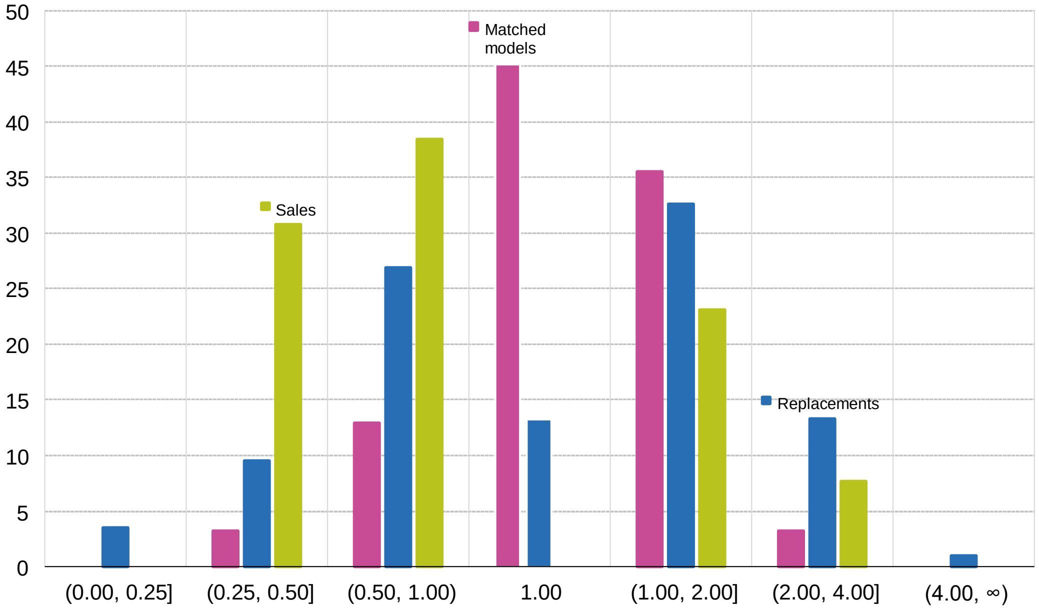
6. For chain drift, the **(high) variance in price relatives in January of each year is decisive**. This variance can be linked to the proportion of within-year (January-to-January) replacements with the coefficient of correlation at 0.63.

Number of price quotes, proportion of within-year replacements (in %)
and variance of the natural logarithms of price relatives (right axis)



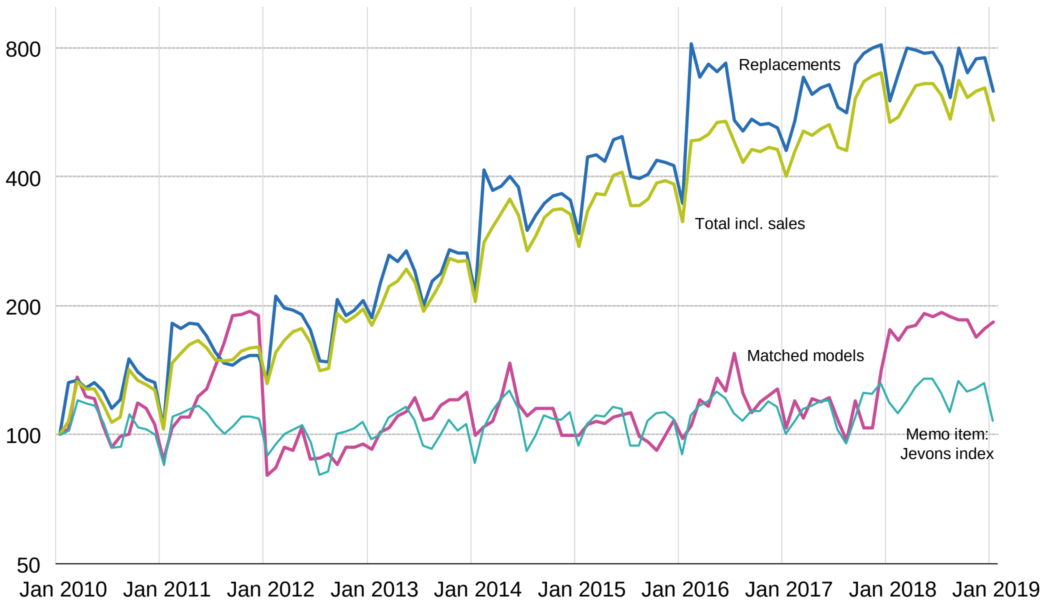
7. Over the nine years under consideration, 24 % of price quotes are flagged as comparable replacement in each month. Over the course of 13 months, this adds up to, on average, 88 % of products being replaced at least once in January of each year.
8. Since replacements are deemed to be comparable, it would to be expected that the **distribution of price relatives for the (at least once) replaced products** does not differ significantly from the one of matched models. However, this hypothesis has to be rejected.

Distribution of price relatives in January, 2011 – 2019
in %



9. The fact that the distribution of price relatives of replaced products has 'fatter' tails and is, on average, shifted to the right compared to the one of matched models is also reflected in the **price indices for these breakdowns**.

Carli price indices for women's vests/strappy tops
Jan 2010 = 100, log scale

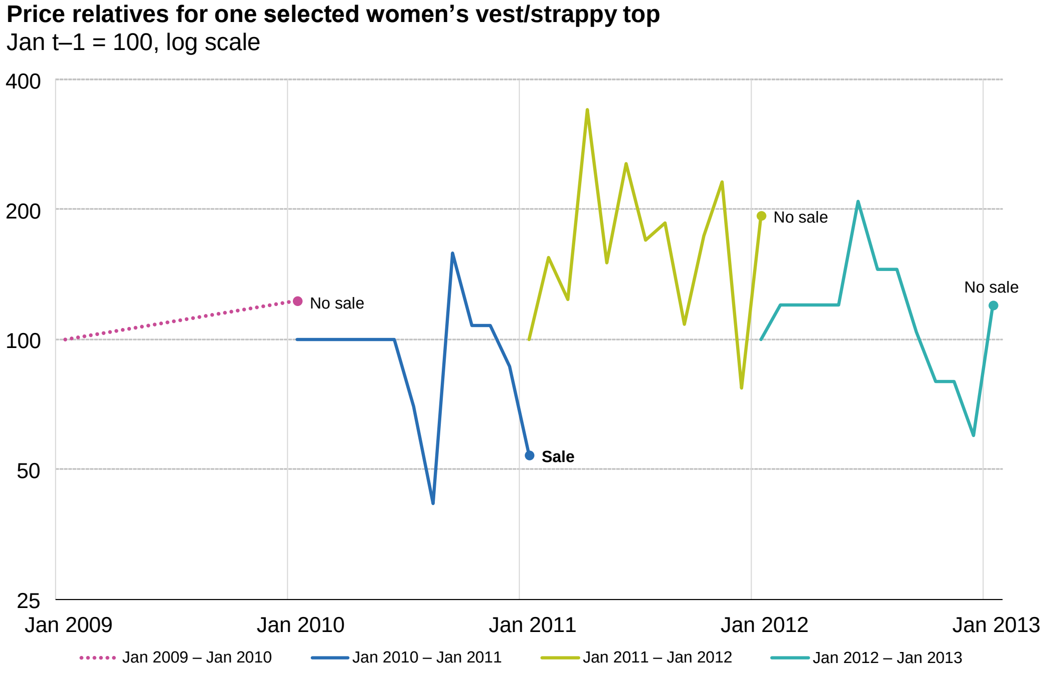


Annex B: On the difference between the direct Carli and chained Carli indices

1. Since one could argue that the comparison of chained Carli to chained Jevons is not the same, the analysis is extended to the comparison of direct Carli to chained Carli.
2. The indefinite bias of the chained Carli index vis-à-vis the direct Carli index depends on the **covariance between the current period and previous period price relatives**. For notational simplicity, 0 refers to January 2010, 1 to January 2011, etc. in what follows.

$$\begin{aligned} P_C^{0,2} - CP_C^{0,2} &= P_C^{0,2} - P_C^{0,1} \cdot P_C^{1,2} \\ &= \mathrm{E}\left[\frac{p_k^1}{p_k^0} \cdot \frac{p_k^2}{p_k^1}\right] - \mathrm{E}\left[\frac{p_k^1}{p_k^0}\right] \cdot \mathrm{E}\left[\frac{p_k^2}{p_k^1}\right] \\ &= \mathrm{Cov}\left[\frac{p_k^1}{p_k^0}, \frac{p_k^2}{p_k^1}\right] \end{aligned}$$

3. **Bias is upward**, i.e. the chained Carli index is greater than the direct Carli index, **whenever the covariance is negative**. The covariance is negative if, for example, sales prices return to their normal levels or vice versa.
4. The calculations require matched models, i.e. no replacements. While this is not feasible for all products in the CPI microdata over the entire period, products can be identified that are **available in two or more consecutive Januarys**.
5. In 29 % of the cases where such products can be identified, sales prices return to their normal levels or vice versa.



6. The problem is elevated by the fact that for chained Carli indices the **bias introduced is cumulative**, hence introducing drift.
7. Quasi-Taylor series approximations reveal the recursive nature of the bias at later points in time; the approximations are 'quasi' since auto-covariances at lags higher than one are ignored.

$$\begin{aligned} P_C^{0,3} - CP_C^{0,3} &\approx P_C^{2,3} \cdot \left(P_C^{0,2} - CP_C^{0,2}\right) + CP_C^{0,1} \cdot \mathrm{Cov}\left[\frac{p_k^2}{p_k^1}, \frac{p_k^3}{p_k^2}\right] \\ P_C^{0,4} - CP_C^{0,4} &\approx P_C^{3,4} \cdot \left(P_C^{0,3} - CP_C^{0,3}\right) + CP_C^{0,2} \cdot \mathrm{Cov}\left[\frac{p_k^3}{p_k^2}, \frac{p_k^4}{p_k^3}\right] \end{aligned}$$

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