ADVISORY PANEL ON CONSUMER PRICES – TECHNICAL

The winning formula? A framework for choosing an appropriate index method for use on web scraped and scanner data

Status: Work in progress

Expected publication: Alongside minutes

Purpose

1. A framework that assesses index number methods against pre-determined quality criteria has been developed to ensure that the methods chosen to produce UK consumer price statistics best meet user needs. Available index number methods have recently been assessed against this framework to produce a shortlist of appropriate methods for use in UK consumer price statistics.

Actions

2. Members of the Panel are invited to:
    a. comment on the questions and scoring system used within the framework to assess the appropriateness of elementary aggregate methods
    b. advise on any competing literature with regards to the assessments that have been made for index number methods within the framework
    c. comment on our plans for future work

Introduction

3. Paper APCP-S(18)03 set out a high-level vision for the Consumer Prices Data Collection Strategy, focusing on a move towards more automated forms of data collection. We proposed four key areas of further work required: obtaining robust sources of alternative data, methods research, assessing the impact on consumer price statistics and the development of systems to support the inclusion of new data sources.

4. In May 2019, we presented APCP-T(19)09; Shortlisting appropriate index methods for use on web scraped and scanner data to the Technical Panel. This discussed the appropriateness of elementary aggregate (EA) methods when applied to alternative data sources and highlighted our chosen criteria as important properties a method should possess. On the advice of the APCP-T, we have since discussed these criteria with the APCP-Stakeholder and applied weights to reflect the relative importance of each criterion. Table 1 provides the criteria and their respective weights. The framework is detailed further in **Annex A**.

Table 1: Criteria weights within the Index Number Methods Framework.

Criterion	Weight
a. Theoretical properties	55%
b. Resource	20%
c. Interpretability	15%
d. Flexibility	10%
e. Coherence	0% (used as a secondary filter)

5. Index number methods can be grouped into weighted and un-weighted formulae depending on whether quantities have been observed alongside prices (quantity data are needed to derive expenditure shares, with which price changes are weighted together in weighted indices). They can also be categorised as being either bilateral or multilateral. Bilateral indices consider price changes between two time periods, although these are not necessarily consecutive. By comparison, multilateral indices make use of three or more time periods simultaneously in the index formula, and typically make use of data in the intermittent time periods.

6. This paper utilises the framework to assess over 50 combinations of methods and extension methods, as shown in Table 2 and described further in **Annex B**.

Table 2: Index number and extension methods under consideration

Bilateral methods[1]	Multilateral methods	Extension methods
Arithmetic Laspeyres	GEKS-J	Direct extension (DE)
Geometric Laspeyres	GEKS-T	Movement splice (MS)
Paasche	GEKS-F	Window splice (WS)
Fisher	Geary Khamis (GK)	Half window splice (HWS)
Törnqvist	Time product dummy (TPD)	Geometric mean splice (GMS)
Jevons	Time product dummy hedonic (TPH)	Fixed base monthly expanding window (FBME)
Dutot		
Carli		

1. Bilateral methods are considered in both their direct and monthly-chained forms

7. A single method may not be suitable for all data sources and commodity groups; therefore, we intend to create a shortlist of appropriate methods rather than simply taking the method with the highest rank. An investigation of the effects of different pricing behaviours on the indices produced by the shortlisted methods is proposed in the future work section of this paper.

Assessment of methods against criteria

8. Each method was assessed against each criterion using a pass/fail system of sub-criteria questions to produce the criterion score; the four weighted criterion scores have been added together to get each method's final score. The final score is used to rank the methods and produce the ONS shortlist of appropriate index methods for consumer prices. In cases of equal scores between methods, the cohesion criterion was used as a secondary filter to separate methods in the rankings. For example, if two methods received the same score in the rankings, any method in use by other NSIs would take precedence in the shortlist.

9. The sub-criteria questions of each criterion are explained, methods that fail to pass are highlighted and reasoned.

a. Theoretical Properties (including characteristicity and transitivity)

10. Given that ONS are primarily concerned with producing accurate and reliable statistics, the theoretical properties criterion was given the highest weight, 55%, and can be broken down into three key areas:

    i. the axiomatic/test approach (30%)

ii. transitivity (15%)

iii. characteristicity (10%)

Transitivity and Characteristicity have been grouped into the Theoretical Properties criterion as all measures are concerned with statistical accuracy.

i. The axiomatic/test approach

11. There are already a known set of tests to measure a method's suitability from a theoretical stand point, known as the axiomatic/test approach. Understanding of the axiomatic approach can be found in the ILO manual (Chapter 16) for bilateral index methods, and in Diewert's chapter in International and Interarea Comparisons of Income, Output, and Prices, for multilateral methods.

12. There is an argument to be made that not all the axioms are as important as each other and, in principle, this presents an aggregation problem of its own, that is how to weight the different axioms. However, for the purpose of this framework, each of the axioms were given an equal weight. The scores for each method can be found in **Annex C**. For the bilateral methods, Fisher scores the highest and Törnqvist the lowest. For the multilateral methods, GEKS-T, GEKS-F and Geary-Khamis score the highest equally, while the hedonic approaches score the lowest.

13. There are two other approaches that are often considered when theoretical properties are discussed, the stochastic approach and the economic approach. The economic approach was given a zero weight within this framework owing to the fact that we do not currently consider a Cost of Living Index (COLI) our primary target[1]. The stochastic approach was considered but also given zero weight for the reasons highlighted in section 6 of Diewert's "Note on the Stochastic Approach to Index Numbers".

ii. Transitivity

14. Transitive indices are free of chain drift, that is they are directly equivalent to the direct, or bilateral index calculated between the start and end periods. In the static-universe, a lack of transitivity is not overly important since chaining is not required. However, transitivity is an important consideration in a dynamic-universe context. High frequency chaining can better account for dynamic prices and high churn but can also result in significant chain drift as a result of non-symmetric effects on quantities sold and expenditure shares before and after products are on sale. Some dynamic methods can be transitive, including some weighted multilateral indices in their purest form.

15. Within our framework, transitivity was measured over two sub-questions, as displayed in Table 3.

Table 3: Assessment of index number methods against the transitivity sub-criteria

Weight	Question	Sub-criteria question	Failing methods (with extensions)
5%	TP4	Is the method transitive across the whole of the time frame?	Paasche (Fixed base & Chained) Arithmetic Laspeyres (Fixed base & Chained) Fisher (Fixed base & Chained) Törnqvist (Fixed base & Chained) Carli (Fixed base & Chained) Geometric Laspeyres (Fixed base & Chained)

[1] The economic approach considers how well an index number method approximates the consumers' cost of living. The CPIH and CPI are not intended to approximate cost of living indices.

10%	TP5	Is the method transitive across a given time window (13 months)?	Paasche (Fixed base & Chained) Laspeyres (Fixed base & Chained) Fisher (Fixed base & Chained) Törnqvist (Fixed base & Chained) Carli (Fixed base & Chained) Geometric Laspeyres (Fixed base & Chained) GEKS-J (FBME) GEKS-T (FBME) GEKS-F (FBME) TPD (FBME) TPH (FBME)

16. The reason that FBME extension causes the methods (other than GK) to fail TP5 is because the only time it uses a full 13 months of data is in the final month of each year. You could say that each month of data over a year becomes more transitive, but less characteristic of the binary comparisons.

17. GK is the exception that has been given a pass using a FBME extension. This is because, by design, the GK compares each period to a base period, rather than to each intermittent period. Therefore, the expanding window will not alter the transitive nature over the 13-month window.

iii. Characteristicity

18. Characteristicity requires transitive multilateral comparisons between member periods to retain the essential features of the binary comparisons that existed between them before transitivity. Prices from distant periods may unduly influence multilateral comparisons and we feel that the impact of these influences should be kept to a minimum when they are introduced into the binary comparison. For example; any multilateral method using a window of nine quarters would have data from over two years previous feeding into its index calculation. The resulting index should be more characteristic of the price movements that occurred within that month, and less characteristic of price movements observed two years ago. Characteristicity was measured over two sub-questions as displayed in Table 4.

Table 4: Assessment of index number methods against the characteristicity sub-criteria

Weight	Question code	Sub-criteria question	Failing methods (with extensions)
2%	TP6	Is the price comparison accurate with the binary time periods being compared?	GEKS-J (DE, MS, WS, HWS, GMS) GEKS-T (DE, MS, WS, HWS, GMS) GEKS-F (DE, MS, WS, HWS, GMS) GK (MS, WS, HWS, GMS) TPD (DE, MS, WS, HWS, GMS) TPH (DE, MS, WS, HWS, GMS)
8%	TP7	Is the price comparison accurate with the binary time periods being compared across the time window (13 months)?	

19. The reason each multilateral method fails using one of the four splicing extension methods (Movement, Window, Half-Window, Geometric Mean), is because each new period index is spliced on to the previous period's price index, and that price index was spliced on to the

period before that's index. So essentially there will always be some contribution of a distant price movement affecting the current index. Although as time moves forward any affect will become more and more diluted by the continued splicing.

20. More important is that characteristicity is present over the chosen window and provided that the window length is suitable and not overly long, each splicing extension method will reflect the binary comparisons between periods.

21. The FBME extension approach calculates indices with respect to a base month so will reflect the binary period comparisons.

b. Resource

22. This criterion was given a weight of 20%, it considers each method with the aim of making the most effective use of alternative data in our suite of consumer prices. A change of method could also offer the chance for automating routine manual processes and improving the use of human resources, making it more viable to produce more frequent or timely outputs.

23. Although resource has a relatively low weight within the framework, any methods that would hinder the timeliness or frequency of UK consumer price indices will not be considered. The resources criteria were measured over four sub-questions, as displayed in Table 5.

Table 5: Assessment of index number methods against the resources sub-criteria

Weight	Question code	Sub-criteria question	Failing methods (with extensions)
8%	R1	Can this method script be maintained without regular human input?	
ESSENTIAL	R2	Are the information processing requirements reasonable and manageable? *– Method is no longer considered if fails.*	GEKS-J (DE) GEKS-T (DE) GEKS-F (DE) TPD (DE) TPH (DE)
4%	R3	Can we scale up the amount of information used without scaling up manual effort?	
8%	R4	Can we scale up the amount of information used within the ONS infrastructure?	

24. Each of the resource's sub-questions were assessed for each method based on the expected size and dimensions of the input data to be used, and the current capabilities within our infrastructure. Should either of these alter, each method should be reassessed. Given the current structure of the scanner and web-scraped data sets, and the infrastructure available to compute the index, each method passes R1, R3 and R4.

25. It is worth acknowledging bilateral methods will be less intensive than multilateral methods as they require less periods of data to compute the statistical output. Methods that use expenditure data (weighted methods) and/or product characteristics (hedonic methods), will also place higher burden on information resources due to the increase of variables used in the computation of the index. Simply put, as the input data sets becomes larger or more complex,

in either number of variables or number of products, these more complex methods will sooner find computational problems (slow processing speed and errors).

26. R2 has been deemed as an essential quality for any method to be chosen. All multilateral methods in combination with the direct extension approach failed this question. This is because the methods would not be producible on a monthly basis; a full year of data is required to make each month's calculation, so it is not until December in each year that you could produce January's index. The FBME will produce timely monthly indices whilst still adopting the same direct approach to annual chaining.

c. Interpretability

27. The interpretability criterion has been given a weight of 15%. It is vital for all statistical agencies to be transparent with the statistics that they produce, and justify the chosen methods used in published statistics. The interpretability criterion assesses how easily understood each method is for users of consumer prices indices.

28. While interpretability has a relatively low weight within the framework, it is deemed essential that the price movements calculated are understood by those involved in the implementation and those who use the output data. Any methods that do not meet this, as a minimum requirement, are not considered further within the assessment. The interpretability criteria were measured over three questions, as displayed in Table 6.

Table 6: Assessment of index number methods against the resources criteria

Weight	Question code	Sub-criteria question	Failing methods (with extensions)
10%	I1	Is the method recognised and well represented in the ILO manual or other trusted literature?	GEKS-J (HWS, FBME) GEKS-T (HWS, FBME) GEKS-F (HWS, FBME) GK (DE, MS, WS, HWS, GMS) TPD (DE, FBME, MS, HWS, GMS) TPH (DE, FBME, MS, WS, HWS, GMS)
ESSENTIAL	I2	Are the price movements calculated understood by those involved in the implementation, and/or those who use the output data? – *Method is no longer considered if fails.*	
5%	I3	Can the method be easily explained to a non-technical audience?	GEKS-J (WS, HWS, GMS) GEKS-T (WS, HWS, GMS) GEKS-F (WS, HWS, GMS) GK (WS, HWS, GMS) TPD (DE, FBME, MS, WS, HWS, GMS) TPH (DE, FBME, MS, WS, HWS, GMS)

29. The bilateral methods and the more established multilateral methods are recognised in the ILO manual and other significant literature. TPD and TPH are relatively new approaches, both considered as more product detail is collected than previously. While more literature is being produced on these hedonic methods, the focused research and implementation by NSIs has been predominantly on the TPD with a window splice, also knows as the fixed effects window splice (FEWS) approach.

30. Similarly, GK has been shown to produce reliable indices with the FBME extension method, as suggested by A.Chessa (A Comparison of Price Index Methods for Scanner Data, 2017), and as such there is little literature found, to date, that suggests alternative extension methods with GK.

31. The GEKS multilateral methods appear most regularly in the ILO manual and trusted research papers. Although due to the inconsistency of the balance between transitivity and characteristicity as each month is added, GEKS is not documented combined with FBME approach.

32. It was considered that interpretability for TPD and TPH may be more difficult for those from a non-statistical background due to the regression approach used to produce the statistical relationships and estimate decomposition.

d. Flexibility

33. This criterion has weight of 10%. It is advantageous for an index method to be flexible enough to be used for a range of purposes, data sources and item types. The flexibility criterion assesses how methods can be used for new products and data sources.

34. Flexibility has a small weight within the framework because if the method cannot adapt to the addition or removal of products, there are additional options to ensure that these behaviours can be accounted for. For example, we can look at following groups of products over time rather than following individual products over time. This would allow product churn to be controlled for without necessarily needing a flexible index number method.

35. It also is not a primary concern if the method is not flexible enough to be used for both scanner data (including quantity information) and web-scraped data (excluding quantity information) as different methods can be used for different data sources and weighted together, or expenditure approximations (using page rankings, for example) could be used.

36. The flexibility criteria were measured over three questions, as displayed in Table 7.

Table 7: Assessment of index number methods against the flexibility criteria

Weight	Question code	Sub-criteria question	Failing methods (with extensions)
3.33%	F1	Does the method adapt to removal of old products?	
3.33%	F2	Does the method adapt to addition of new products?	Paasche (Fixed base) Arithmetic Laspeyres (Fixed base) Fisher (Fixed base) Törnqvist (Fixed base) Jevons (Fixed base) Dutot (Fixed base) Carli (Fixed base) Geometric Laspeyres (Fixed base) GEKS-J (DE, FBME) GEKS-T (DE, FBME) GEKS-F (DE, FBME) TPD (DE, FBME) TPH (DE, FBME)
3.33%	F3	Does the method utilise available quantity data?	Jevons (Fixed base & Chained) Dutot (Fixed base & Chained) Carli (Fixed base & Chained) GEKS-J (DE, FBME, MS, WS, HWS, GMS)

37. There are many benefits to using a fixed base approach to calculating price indices however one drawback is that, without manual intervention, new products can only be introduced to the basket in the first period after rebasing. Therefore, all methods that make use of a fixed base fail F2.

Results - Index methods shortlisted for use on dynamic data sets

38. Two shortlists have been created using the framework described within this paper; one for data that has quantity information available (or approximated) and one where no quantity data are available, and quantities are unable to be approximated.

39. Table 8 provides the shortlisted methods for data where quantity information (whether actual or approximated) is available.

Table 8. Shortlisted methods for dynamic data with quantity information

Method	Extension	Rank	Score	Uses Quantity Data
Geary-Khamis	FBME	1	0.95	Yes
GEKS-Törnqvist	Movement Splice	2	0.93	Yes
GEKS-Fisher	Movement Splice	3	0.93	Yes
Jevons	Chained Monthly	4	0.90667	No
Dutot	Chained Monthly	5	0.88667	No
GEKS-Jevons	Movement Splice	6	0.88667	No
GEKS-Törnqvist	Geometric Mean Splice	=7	0.88	Yes
GEKS-Fisher	Geometric Mean Splice	=7	0.88	Yes
GEKS-Törnqvist	Window Splice	=7	0.88	Yes
Time Product Dummy	Window Splice (FEWS)	17	0.83	Yes

40. Although the method did not rank within the top 10 on the shortlist, FEWS was also added to the shortlist using the cohesion criteria. It was the highest scoring approach that utilises hedonic pricing, and is widely acknowledged, if not used, by NSIs for scanner data sets in consumer statistics.

41. GEKS-Törnqvist with a movement splice (RYGEKS-T) and GEKS-Fisher with a movement splice (RYGEKS-F) scored equally in the shortlist. Again, the cohesion filter was used to rank these methods in the shortlist. Therefore RYGEKS-T has become the second-choice method (as other NSIs have begun to adopt this approach), with RYGEKS-F third-choice. For the same reason, chained monthly Dutot ranks higher than RYGEKS-J.

42. Table 9 provides the shortlisted methods for data where quantity information (whether actual or approximated) is unavailable.

Table 9. Shortlisted methods for dynamic data without quantity information

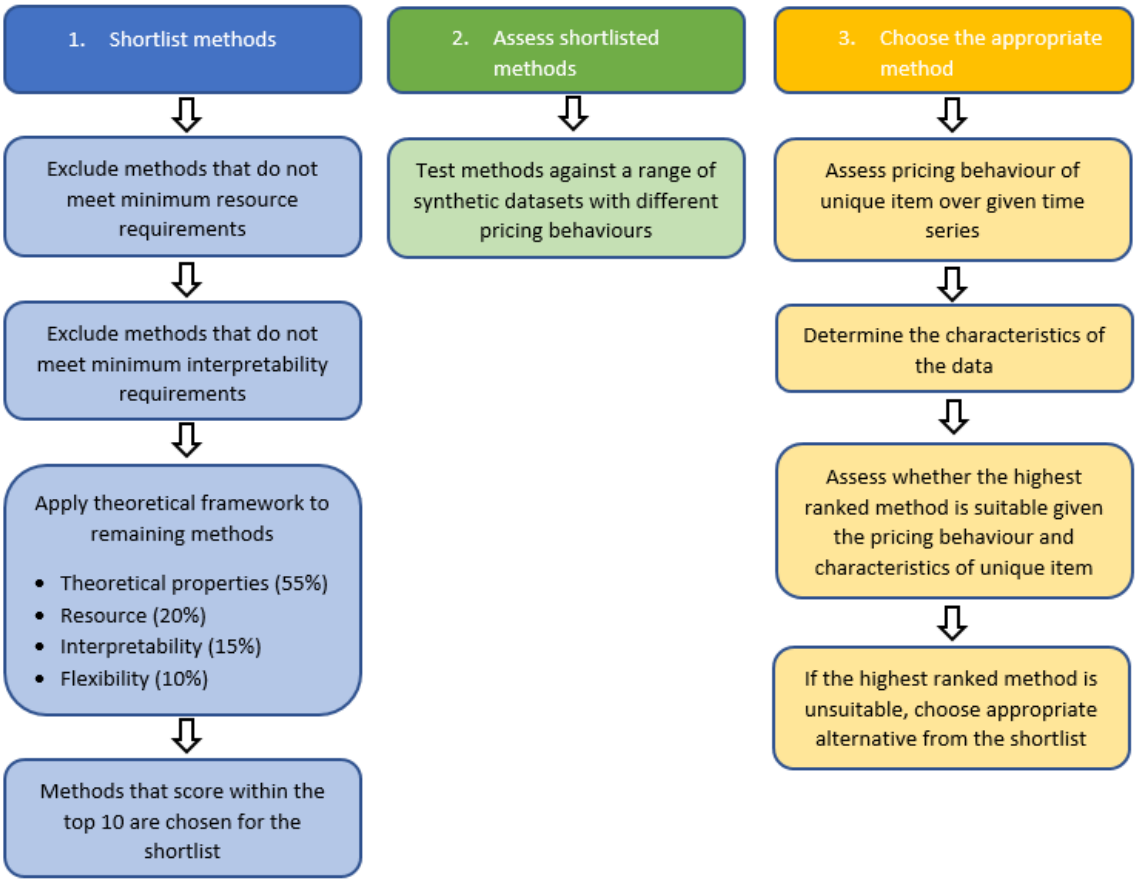
Method	Extension	Rank	Score	Uses Quantity Data
Jevons	Chained Monthly	4	0.90667	No
Dutot	Chained Monthly	5	0.88667	No
GEKS-Jevons	Movement Splice	6	0.88667	No
Jevons	Fixed Base	12	0.87333	No
Dutot	Fixed Base	13	0.85333	No
GEKS-Jevons	Geometric Mean Splice	14	0.83667	No

43. While fixed base Jevons, fixed base Dutot and GEKS-J extended with a geometric mean splice were not included in the earlier shortlist when quantity information is available, the decision was made to include them here to add to the available options when quantity data is unavailable and will provide further comparison with the first-choice approach; chained Jevons.

Future Work

44. A visualisation of the process for choosing an appropriate index method for each unique item is shown in Figure 1. This paper addresses the left side of this figure (in blue), while future work will begin to address the middle (green) and right side (yellow) of the figure.

Figure 1. Process for choosing an appropriate index number method for each unique item



45. The proposed follow on from this paper is therefore to take the shortlisted methods (Tables 8 and 9) and apply them to synthetic data sets that feature certain characteristics (for example, high product churn, high price variance, clearance prices) that could be problematic for an index number method, then suggest an appropriate alternative method from the shortlist, should GK be unsuitable.

46. Future work will include case studies within the areas of the inflation basket that have been predetermined as high priority areas for the use of web scraped or scanner data, and comment on the appropriateness of GK and the shortlisted methods for each item. These items were prioritised with APCP-S in September 2019, and include technological goods, package holidays, clothing, used cars, chart collected items (CDs, DVDs, Books), airfares, rail fares and groceries.

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List of Annexes

Annex A	**Summary of criteria & weights**
Annex B	**Technical description of the shortlisted methods**
Annex C	**Axiomatic assessment of bilateral and multilateral methods**

Annex A – Summary of criteria and weights

1. Our guidelines for measuring statistical quality outline best practice for measuring and reporting on the statistical quality of outputs. These guidelines, in combination with the comprehensive literature review carried out in APCP-T(19)09, and input from technical experts and consumer prices stakeholders has led to the following criteria and associated weights.

Resources - 20%

2. The resources criteria would fall into the timeliness dimension of the ONS quality criteria and aims to answer the question "does this method enable more effective use of human and information resources?".

3. In terms of computing requirements and processing power needed, bilateral methods will be less intensive than multilateral methods as they require less periods of data to compute the statistical output. Methods that use expenditure data (weighted methods) and/or product characteristics (hedonic methods), will also place higher burden on information resources due to the increase of variables used in the computation of the index.

Theoretical Properties – 55%

4. This criterion belongs to the accuracy and reliability dimension and can be broken down into three sub criteria; axiomatic approach to index numbers (30%); transitivity (15%); and characteristicity (10%).

5. The axiomatic/test approach is where the index is tested against some desirable properties (see International Labour Organisation (ILO) consumer price index manual chapter 16).

6. Price indices possess the transitive property when they give the same result as an index calculated directly between two periods using the same data. Transitivity is a desirable property for price comparisons because the results will be independent of the choice of base period.

7. Characteristicity is the property that requires transitive multilateral comparisons between member periods to retain the essential features of the binary comparisons that existed between them before transitivity, as prices from distant periods may unduly influence multilateral comparisons.

Flexibility – 10%

8. It is an advantage for an index method to be flexible enough to be used for a range of purposes, data sources and types of items. The flexibility criterion assesses how methods can be used for new products and data sources and falls into the cohesion and comparability dimension.

Interpretability – 15%

9. The accessibility and clarity dimension is relevant to this criterion. It is vital for all statistical agencies to be transparent with the statistics they produce, and justify the chosen methods used in published statistics. There are two aspects to the interpretability criterion; firstly, how easily understood the methods are for users to understand and; secondly, whether the price movements that each index produces are easy to interpret, particularly the products/categories of greatest influence and why.

Cohesion – No Weight, used as a secondary filter

10. Unsurprisingly this criterion coincides with the cohesion and comparability dimension within the guidelines. The cohesion criterion is twofold; internal cohesion across different areas of

the basket, and different data sources and; external cohesion with other NSI to aid comparison between countries.

11. The decision was made, given user feedback, that the cohesion criteria should be used as a secondary filter, to separate equal scoring methods, rather than to contribute to a method score. This is because a method should be chosen for its qualities, not simply because others are using the method. Ideally, should other NSI prioritise similar criteria as ONS, they would reach the same chosen methods for their indices.

Annex B –Technical descriptions of the shortlisted methods

1. The Geary-Khamis (GK) index was developed for PPPs, but unlike the GEKS, which compares each period to each other, the GK index compares each period to a base period. It is an implicit price index that divides a value index by a weighted quantity index. Using notation like Chessa [2016] it is defined as:

$$P_{GK}^{t} = \frac{\frac{\sum_{i \in S^*} p_i^t q_i^t}{t}}{\frac{\sum_{i \in S^*} p_i^0 q_i^0}{\frac{\sum_{i \in S^*} v_i q_i^t}{\sum_{i \in S^*} v_i q_i^0}}} \quad (1.1)$$

where the weights v_i are as follows:

$$v_i = \sum_{z \in T} \phi_i^z \frac{p_i^z}{P_{GK}^z}$$

$$\phi_i^z = \frac{q_i^z}{\sum_{s \in T} q_i^s}$$

2. The GEKS-T is calculated as the geometric mean of the ratios of all matched-model bilateral indices $P^{l,t}$ and $P^{l,o}$ where each period, l, is taken in turn as the base. The GEKS-T method can be expressed as:

$$P_{GEKS-T}^{(0,t)} = \prod_{l=0}^{T} (P_T{}^{0,l} P_T{}^{l,t})^{\frac{1}{T+1}} \quad (1.2)$$

3. From the empirical viewpoint it can be expected that the GEKS-Fisher and the GEKS-Törnqvist indices closely approximate each other. The GEKS-F method can be expressed as:

$$P_{GEKS-F}^{(0,t)} = \prod_{l=0}^{T} (P_F{}^{0,l} P_F{}^{l,t})^{\frac{1}{T+1}} \quad (1.3)$$

4. The Jevons index is calculated as a geometric mean of current period price relatives. The Jevons formula is used in over 60% of items in the UK CPI but cannot be used when prices fall to exactly zero. (Paul Johnson review, UK consumer price statistics - Chapter 10, 2015)

$$P_{Je}^{0,t} = \sqrt[n]{\prod_{i=1}^{n} \frac{p_i^t}{p_i^0}} = \sqrt[n]{\prod_{i=1}^{n} R_i^{0,t}} \quad (1.4)$$

5. The Dutot index is calculated as the ratio of average prices in the current and base periods (given a matched basket between both periods) and is usually used for homogeneous items as the formula implicitly gives greatest weight to the highest priced product. The Dutot formula is used for a small number of items in the UK CPI (generally those where the Jevons formula cannot be applied).

$$P_{Du}^{0,t} = \frac{\frac{1}{n}\sum_i^n p_i^t}{\frac{1}{n}\sum_i^n p_i^0} = \sum_i R_i^{0,t} \frac{p_i^0}{\sum_i p_i^0} \quad (1.5)$$

6. There is no justification in favouring taking the expenditure weights from either the base or current periods, hence Fisher proposed taking an average of the Paasche and Laspeyres indices to derive a single measure of price change. Taking a geometric mean leads to the so-called Fisher ideal price index.

$$P_F^{0,t} = \sqrt{P_L^{0,t} P_P^{0,t}} \quad (1.6)$$

Where,

$$P_L^{0,t} = \frac{\sum_{i=1}^n p_i^t q_i^b}{\sum_{i=1}^n p_i^0 q_i^b} (1.7)$$

$$P_P^{0,t} = \frac{\sum_i p_i^t q_i^t}{\sum_i p_i^0 q_i^t} \quad (1.8)$$

7. The GEKS-J price index formula is defined as follows:

$$P_{GEKS-J}^{(0,t)} = \prod_{i=0}^{t} (P_J{}^{0,i}\, P_J{}^{i,t})^{\frac{1}{t+1}} \quad (1.9)$$

8. The TPD aims to decompose the price of a product into how much of the price comes from being that specific product and how much comes from it being observed in a specific time period. The TPD method uses a regression approach that is like those of hedonic based methods - it uses the statistical relationship between prices, products and time to estimate the decomposition. The TPD model is expressed as:

$$\ln p_i^t = \alpha + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{i=1}^{N-1} \gamma_i D_i + \varepsilon_i^t \quad (1.10)$$

where,

$\ln p_i^t$ = log of price for item *i* in period t
α= intercept term
δ^t= time parameter corresponding to time period t
D_i^t= time dummy variable, equal to 1 if the price observation p_i^t was observed to period t and 0 otherwise
γ_i= product parameter corresponding to product *i*
D_i= product dummy variable, equal to 1 if the price observation p_i^t pertains to item *i* and 0

otherwise
ε_i^t= error term

Extension methods

9. The movement splice method involves joining a new period into a new multilateral comparison window and extending the index based on a price comparison from this new window. The index level in this new period is calculated by multiplying the previous published index level by the price movement between the previous and the new period, as estimated using the new multilateral window (Ivancic, Fox and Diewert 2011). This is equivalent to the approach used to produce chained bilateral indices and can be expressed as:

$$P_{MS}^t = P^{t-1} \times P^{t-1,t}(t-T) \quad (1.11)$$

where,

P_{MS}^t= index level in period *t*
P^{t-1}= index level in the previous period
$P^{t-1,t}(t-T)$ = price movement between *t-1* and *t* using the latest multilateral window (of length T) to generate a price comparison between *t-1* and *t.*

10. The window splice method, proposed by Krsinich (2016), uses the rolling window approach to extend the index when a new period of data is available, similarly to the movement splice. However, the methods use price movements from the latest multilateral comparison window to update the index differently. Whereas the movement splice method joins the last period-on-period movement from this window, the window splice method joins on the latest full window onto the index level of *T* periods earlier. This can be expressed as:

$$P_{WS}^t = P^{t-1} \times \frac{P^{t-T,t}(t-T)}{P^{t-T,t-1}(t-T-1)} \quad (1.12)$$

where,

P_{WS}^t= index level in period *t*
P^{t-1}= index level in the previous period
$P^{t-T,t}(t-T)$= price movement between *t-T* and *t* using the latest multilateral window between *t-T* and *t*
$P^{t-T,t-1}(t-T-1)$= price movement between *t-T* and *t-1* using the previous multilateral window between *t-T-1* and *t-1*.

11. FBME uses a time window with a fixed base month, which is shifted each year to the next base month. To include data from a new month the time window is extended with each month. To ensure price indices are free of chain drift the indices are calculated with respect to the base month with the most recent set of parameter values. FBME can be expressed as:

$$P_{FBME}^t = P^{(0,t)} \quad (1.13)$$

where,
P_{FBME}^{t}= Price index in period *t*
P^{0}= Price index level in the base period

12. Diewert and Fox (2017) propose the use of a "mean splice" by taking the geometric mean of all the price indices that are obtained using every possible link period, given the window length.
The ABS have identified some issues with movement and window splicing and highlight the use of mean splice instead; the movement splice can yield downward drift due to disappearing items with unusually low prices whereas the window splice can yield downward drift due to new items entering with unusually high prices. However, the mean splice acts more like a movement splice near the start of the window and more like a window splice near the end; mitigating problems with disappearing and new items.

Annex C – Axiomatic assessment of bilateral and multilateral methods

Reference	Bilateral Axiom Property	Paasche	Laspeyres	Fisher	Törnqvist	Jevons	Dutot	Carli	Geo Laspeyres
TP1	Positivity	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TP2	Continuity	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TP3	Identity	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TP4	Fixed Basket Test	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
TP5	Proportionality in current prices	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TP6	Inverse Proportionality in base prices	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TP7	*Invariance to proportional changes in current quantities*	Yes	Yes	Yes	Yes	N/A	N/A	N/A	Yes
TP8	*Invariance to proportional changes in base quantities*	Yes	Yes	Yes	Yes	N/A	N/A	N/A	Yes
TP9	Commodity reversal	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TP10	Invariance to change in units of measurements	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes
TP11	Time reversal test	No	No	Yes	Yes	Yes	Yes	No	No
TP12	*Quantity reversal test*	No	No	Yes	No	N/A	N/A	N/A	No
TP13	Price reversal test	No	No	Yes	No	No	No	No	No
TP14	Mean value test for prices	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TP15	*Mean value test for quantity*	Yes	Yes	Yes	No	N/A	N/A	N/A	Yes
TP16	Paasche and Laspeyres bounding test	Yes	Yes	Yes	No	No	No	No	Yes
TP17	Monotonicity in current prices	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
TP18	Monotonicity in base prices	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
TP19	*Monotonicity in current quantities*	Yes	Yes	Yes	No	N/A	N/A	N/A	Yes
TP20	*Monotonicity in base quantities*	Yes	Yes	Yes	No	N/A	N/A	N/A	Yes
TP21	Additivity	Yes	Yes	No	No	No	No	No	Yes
	Bilateral Axiom Score	0.257143	0.257143	0.285714	0.157143	0.24	0.22	0.22	0.257143

Axiom Reference	Multilateral Axiom Property	GEKS-J	GEKS-F	GEKS-T	Geary-Khamis	Time Product Dummy	Time Product Hedonic
TP22	Share Test	Yes	Yes	Yes	Yes	Yes	Yes
TP23	*Proportional Quantities Test*	N/A	Yes	Yes	Yes	No	No
TP24	Proportional Prices Test	Yes	Yes	Yes	Yes	Yes	Yes
TP25	Commensurability Test	Yes	Yes	Yes	Yes	Yes	Yes
TP26	Commodity Reversal Test	Yes	Yes	Yes	Yes	Yes	Yes
TP27	Multilateral period reversal test**	Yes	Yes	Yes	Yes	Yes	Yes
TP28	Monetary Units Test	Yes	Yes	Yes	Yes	Yes	Yes
TP29	*Homogeneity in Quantities Test*	N/A	Yes	Yes	No	Yes	Yes
TP30	Monotonicity Test**	Yes	Yes	Yes	No	Yes	Yes
TP31	Partitioning Test**	No	No	No	Yes	No	No
TP32	Bilateral Consistency in Aggregation	Yes	Yes	Yes	Yes	No	No
TP33	Additivity**	No	No	No	Yes	No	No
	Multilateral Axiom Score	0.24	0.25	0.25	0.25	0.2	0.2

**These properties may not hold after extension

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