**Coverage Estimation Strategy for the 2021 Census of England & Wales**

**Version 1.7**

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1. **Introduction**

It is a well-known fact that even an exceptionally well designed and implemented census of a large and complex population cannot provide an enumeration with ignorable coverage errors [Marks *et al.*, 1953; National Research Council, 1984; Brown *et al.,* 1999]. Furthermore, it has been long ago established that the coverage errors tend to be non-uniformly distributed across a population [Price, 1947; Coale, 1955; Siegel 1974; Robinson *et al.,* 1993].

Census coverage estimation (CCE) refers to a set of statistical methods for estimating census coverage related parameters as well as some tools and processes enabling implementation of such methods. The purpose of CCE in England & Wales (E&W) is to estimate the census coverage errors and the coverage error corrected population size (along with the corresponding uncertainty measures) at national and local authority (LA) levels and lay out the basis for replacing estimated census omissions with synthetic records in the census output database.

The CCE strategy is divided into four key components: estimation of the undercoverage errors (caused by units of a population missing from a census) together with estimation of the variance of undercoverage errors; estimation of the overcoverage errors (caused by units of a population counted more than once and units of a population counted in a wrong location); residual bias adjustment and national adjustment (correcting for some of the residual errors caused by factors such as the dependency between census and the census coverage survey (CCS), drawing a non-representative sample in an area, etc.); and the coverage estimation of communal establishments (CE).

Some of the critical coverage estimation enabling processes such as the design of the CCS and census to CCS linkage are being developed in workstreams of their own and are discussed separately [Castaldo, 2018a]. The focus of this paper is the methods for coverage estimation. Throughout this paper it is assumed that all the processes the CCE relies on such as the CCS design and data collection along with the record linkage are implemented to the highest standards required by the CCE. The key requirements include independence between the Census and CCS, high CCS response rate (approximately 90% nationally, with 95% of LAs having response rate over 80% and no LA with the response rate below 70%) [Castaldo, 2018b] and nearly perfect CCS to Census linkage. Any major flaws in the above processes would nullify the chances of the CCE achieving its goals.

1. **Coverage estimation in the 2011 Census of E&W**

In many respects, the 2011 CCE was a continuation of the seminal 2001 One Number Census (ONC) project [Brown *et al.*, 1999; ONS, 2001a; ONS, 2001b]. The key ambition of the ONC was to produce a high quality national population estimate simultaneously allowing all possible census tabulations to add to that national estimate.

In a similar fashion to the 2001 ONC, in 2011 the CCS provided a census independent enumeration of a sample of areas. The sample size was approximately 340k households (equivalent to 1.5% of postcodes) spread across all LAs in E&W [ONS, 2012a; ONS, 2012b]. In terms of sample design, the 2011 CCS was a stratified (LA by hard-to-count (HtC)) two-stage cluster (output areas, postcodes) sample as described by Brown *et al.* (2011). Once the survey data were collected, responding households and individuals were linked to the census data to make the cells required for the dual system estimation (DSE) observable [Sekar and Deming, 1949; Wolter, 1986; for more general discussion see Bishop *et al.* 1975; Seber, 1982; McCrea and Morgan, 2015]. DSE was used as the basis for the estimation strategy as it estimates the total population in the presence of non-response, given certain assumptions.

The DSE for individuals (1) was post-stratified by LA, HtC, a cluster of postcodes and age-sex group to provide protection against heterogeneous response propensities.

$$\begin{array}{c}\hat{t}\_{alhc}= \frac{\left(X\_{alhc}+1\right)\left(Y\_{alhc}+1\right)}{\left(M\_{alhc}+1\right)}-1,\#(1)\#\\X\_{alhc}- census count for age-sex a, in LA l, HtC h, cluster c; Y\_{alhc}-corresponding survey count; \\\#\\M\_{alhc}- corresponding census to survey match count.\end{array} $$

The ratio estimator (2) [Royall 1970; Valliant *et al*., 2000] was used in combination with the DSE (3) to estimate the overall population size.

$$E\left(Y\_{alhs}|X\_{alhs}\right)= R\_{alhs}X\_{alhs},$$

$$Var\left(Y\_{alhs}|X\_{alhs}\right)= σ\_{alh}^{2}X\_{alhs},$$

$$\begin{array}{c}Cov\left(Y\_{alhs\_{1}}, Y\_{alhs\_{2}}|X\_{alhs\_{1}}, X\_{alhs\_{2}}\right)=0, ∀ s\_{1}\ne s\_{2, }\#\left(2\right)\end{array}$$

$$\hat{R}\_{alh}= \frac{\sum\_{s\in S\_{lh}}^{}Y\_{alhs}}{\sum\_{s\in S\_{lh}}^{}X\_{alhs}},$$

$$X\_{alhs}- census count for age-sex a, in LA l, HtC h, area s; Y\_{alhs}-corresponding survey count.$$

$$\begin{array}{c}\hat{T}\_{alh}= \frac{\sum\_{c\in S\_{lh}}^{}\hat{t}\_{alhc}}{\sum\_{s\in S\_{lh}}^{}X\_{alhs}}X\_{alh}. \#\left(3\right)\end{array}$$

Since the majority of LAs (approximately 90%) did not have a sample size large enough to produce direct population estimates using (3), the sample-poor LAs were pooled together to form Estimation Areas (EA). This pooling was based on the expected similarity of LAs in terms of the coverage pattern and geographical proximity of the areas [ONS, 2011a]. There was also an operational reason to batch the data in this way to pave the way for early data processing on the batches. The resulting size of EAs was 0.5 million individuals on average. The Ratio estimator was post-stratified by EA, HtC and age-sex. The default option for the ratio estimation was a direct model-based approach. However, there was an allowance for direct model-assisted estimation (4) [Sarndal *et al.,* 1992] for the cases with differential sampling fractions for the strata within an EA *and* evidence of localised census coverage problems.

$$\begin{array}{c}\hat{R}\_{aeh}= \frac{\sum\_{s\in S\_{eh}}^{}w\_{eh}Y\_{aehs}}{\sum\_{s\in S\_{lh}}^{}w\_{eh}X\_{aehs}}, \#\left(4\right)\end{array}$$

$$X\_{aehs}- census count for age-sex a, in EA e, HtC h, area s;$$

$$ Y\_{aehs}-corresponding survey count, w\_{eh} - sampling weight.$$

See Brown et al. (2018) for more details. The LA-level estimates were produced by indirect synthetic estimator (5) [Baffour *et al.,* 2018].

$$\begin{array}{c}\hat{T}\_{alh}= \frac{\hat{T}\_{aeh}}{X\_{aeh}}X\_{alh}. \#\left(5\right)\end{array}$$

The broad similarity in the data collection approaches between the 2011 and 2001 Censuses provided the ground for efficient stratification and sample allocation. Experience gained in the ONC allowed for better integration of bias adjustments in 2011, for example the methodology described by Brown *et al.* (2006) to measure and adjust for dependence. In addition, a methodology for overcoverage adjustment was developed and introduced [Large *et al.*, 2011]. Arguably, one of the key innovations of the 2011 Census was a better connection between the census data collection and coverage estimation mainly via minimising the variability in the census response while simultaneously maximising the response rate, described by Abbott and Compton (2014).

The evaluation of the 2011 Census estimation methodology concluded that the coverage estimation methods worked well and the outcomes met prior expectations (ONS, 2013). The key lessons to take away from the 2011 Census estimation work were: early development and testing of the methods and systems (including testing of the system during the Census Rehearsal); having a backup estimation approach ready for the Census in time; delivering a better balance between robustness and smoothness of the age-sex estimates; development of non-symmetric confidence intervals [ONS, 2013; ONS, 2015]. All the above lessons are being addressed in the 2021 CCE.

1. **An overview of the 2021 Census of E&W**

The key difference between the 2021 Census and the previous ones is the prioritisation of online data collection [ONS, 2016]. Online data collection was first introduced in the 2011 Census, but all households on the census address list received a paper census questionnaire so that the respondents had a choice between the paper and online responses. There were 16% of those who responded online. In contrast, it is planned to send paper questionnaires to approximately 10% of households at the start of the data collection and to receive approximately 75% of all census responses online in 2021.

There will be an attempt to optimise the census data collection in 2021. For instance, areas might be classified into groups in order to identify the most efficient approach to collect the census data in each of the groups. It also involves the creation of the response chasing algorithm (RCA) [ONS, 2017a]. The RCA is a tool that compares an actual return rate for an area to a predicted return rate at a particular time of the data collection process and flags if the discrepancy between the actual and predicted values are a cause for concern.

Information on several new topics may be collected in the 2021 Census questionnaire. Among the variables that are considered for the inclusions are gender identity and sexual orientation [ONS, 2017b]. Such variables may potentially have an impact on patterns of non-response.

1. **Coverage estimation strategy for the 2021 Census**

The main goal of the 2021 CCE is to provide the high-quality coverage error corrected population totals and the variance of the population totals at the national and certain subnational levels. The preliminary quality targets for the population size estimates are as follows: the relative 95% confidence interval for the national population size estimate less or equal to ±0.2%, the absolute relative bias at the national level less than 0.5%; the relative 95% confidence intervals for the LA totals less or equal to ±3.0%.

The 2021 CCE builds on the achievements and insights of the 2011 CCE. As in the previous two censuses, the proposed strategy is to undertake a CCS and the CCS data will be linked to the census data for the under- and overcoverage estimation. There are several areas where gains in accuracy of estimates, processing speed etc. could potentially be achieved. Equally, there are areas where a loss in accuracy may be incurred hence additional work mitigating those losses is needed.

The main risks to the estimation due to changes in the 2021 Census include: increased uncertainty in census data collection process caused by a heavy reliance on the complex planning work of the entire field operations (such as, identification of areas for sending the paper forms first, attempts to accurately predict the respondents engagement with the online services, etc.); inability to rely on the 2011 Census data to the same extent as the 2011 Census relied on the 2001 Census data; mode effects; decreased response rate due to the introduction of additional questions [ONS, 2017c], allowing for individual responses; increased overcoverage rate resulting from potential multiple questionnaire completion. The aim is to design an estimation methodology that is capable to produce the statistics of required quality in the presence of the risks mentioned. However, it is important to remember that the estimation process has limits and work identifying those limits is ongoing. The results of this work are being fed to the census design to inform the data collection [Račinskij, 2015] and data pre-processing.

Many of the potential improvements in the 2021 CCE directly or indirectly arise from the intention that the census data for the entire country are going to be available for the estimation quicker than it was in 2011. This will result from a smaller fraction of paper returns among all returns, and thus less reliance on scanning for data capture. No batching into EAs is needed and the CCE can be carried out either at the national or at the regional levels, similar to the approach used for example in the 2010 US Census [US Census Bureau, 2008; US Census Bureau, 2012]. A sufficiently large sample of data permits the census coverage for a reasonably large set of characteristics to be modelled using logistic or mixed effects logistic regression models [Alho, 1990; Alho *et al.,* 1993; Saei and Chambers, 2003; Chambers and Clark, 2012]. This approach essentially integrates the dual-system, Ratio and Local Authority estimation parts of the 2011 methods. Estimated census non-response weights (reciprocals of estimated census response probabilities) for an individual / household with certain characteristics can then be applied to each census observation with the corresponding characteristics [US Census Bureau, 2012]. Summing up all weighted census observations with the characteristic of interest will produce an estimated population size of units with the characteristic (6), (7).

$$\begin{array}{c}\hat{T}\_{al}^{LR}= \sum\_{r\in al}^{}\begin{array}{c}\left[\frac{1}{1+ e^{\left(-\left[\hat{β}\_{0}+ \sum\_{i=1}^{k}\hat{β}\_{i}x\_{i}\right]\right)}}\right]^{-1}, \#\\\end{array} \#\left(6\right)\end{array}$$

$$\begin{array}{c}\hat{T}\_{al}^{MELR}=\sum\_{r\in al}^{}\begin{array}{c}\left[\frac{1}{1+ e^{\left(-\left[\hat{β}\_{0}+ \sum\_{i=1}^{k}\hat{β}\_{i}x\_{i}+ \hat{u}\_{l}\right]\right)}}\right]^{-1}, \#\\\end{array} \#\left(7\right)\end{array}$$

There are several important gains expected from the modelling approach. Modelling the census coverage at higher levels of geography than EA allows controlling the effects other than age, sex, HtC and geography and therefore deals with heterogeneity bias more efficiently. Our research [Racinskij, 2017] shows that, in the 2011 Census methodology, the lack of control for variables such as ethnicity, tenure, household relationships, etc. could result in relative bias of at least -0.2% at the national level. The magnitude of the bias does not seem large, but given the census target of the absolute relative bias at the national level not to exceed 0.5%, it proves to be worth considering. In addition, some parts of the residual bias adjustment assume that the estimates for females are not affected by the heterogeneous response. However, our research shows that this is not an entirely valid assumption and reducing the heterogeneity for all sexes may increase the quality of the residual bias adjustment.

The second gain is that the coverage modelling does not rely on synthetic assumptions when estimating the LA level population size. The synthetic assumption is that there is a similar undercoverage pattern for a domain of interest in each LA within an EA. The research shows that even though combining the LAs into EAs in 2011 was very successful, presumably there were a number of areas where the synthetic bias was substantial enough to cause the coverage probability of the confidence intervals to be considerably different from the nominal coverage probability. Note, however, that the LA synthetic estimation was the best possible option available in the 2011 Census [Baffour *et al.,* 2018]. Our initial research indicates that the mixed effects logistic regression based method can achieve higher overall accuracy both at the national and subnational levels. Figure 1 presents the relative root mean square errors for LA population size totals across entire E&W (estimated using simulated data). It can be seen, that the estimation method based on the mixed effects logistic regression (dashed orange line) performs as good as or better the DSE / Ratio / Synthetic approach for the majority of LAs. The ‘picks’ in DSE / Ratio / Synthetic are largely attributable to the heterogeneity and synthetic biases. The other advantage is that this can be achieved in a single step, reducing complexity and perhaps increasing transparency. Clearly, the modelling approach is not itself assumption free and careful model selection is required to avoid the model bias.

Consistent population size estimates for a set of variables (age-sex, ethnicity, activity last week, etc.) might then also be available from a single estimation process. Therefore, there is no need for separate estimation for each variable and subsequent calibration to the age-sex estimates under the coverage modelling approach. It is also potentially feasible to get both person and household estimates from the similar and closely related models. Nevertheless, it is unlikely that all variables could be estimated in this integrated spirit. For instance, the household size variable may still require a separate estimation procedure but this may able to be similar to the approach used in 2011.



**Figure 1.** Relative root mean square error (RRMSE) for DSE / Ratio / Synthetic and Mixed effects logistic regression estimation approaches for every LA in E&W

Logistic / mixed effects logistic regression modelling approaches allow having continuous effects. It also allows some flexibility accounting for the outcome of the census data collection process and the outcome of the online / paper data collection. This can be achieved by, say, having the census return rates, paper return rates, online return rates, broadband coverage, population segmentation (or some form of post-segmentation) or interaction between some of the above as certain area-level covariates. These could improve model fit. Nevertheless, it is desirable for the CCS stratification to differentiate between the areas with high / low predicted online participation to ensure that samples of sufficient size are achieved across areas.

It is not straightforward to implement the proposed undercoverage estimation strategy. Among the biggest challenges are ensuring the CCS design is optimized for and appropriately reflected in the modelling and the model is appropriately selected so that the proposed approach results in accuracy gains both at the national and the subnational levels. Current research uses the 2011 undercoverage estimation approach to benchmark the proposed approach. Therefore, knowledge relevant to the 2011 CCE approach is being maintained. The proposal is to keep the 2011 undercoverage estimation approach as a backup option for the undercoverage estimation. In the case of major data collection / processing issues and a strong evidence that the modelling approach would produce poorer quality estimates compared to the 2011 approach, the latter could be used (at least in the areas where major issues occurred). If the backup method is used we would still expect the modelling provide valuable information for the backup version, say, information for more efficient batching of the LAs into EAs. From a practical perspective, we will also ensure we allow time and develop guidelines for fitting such a complex model, as it would not be able to be fully specified in advance (as it was in 2011). The guidance would form part of the methodology and would need to be demonstrated to be robust, and acceptable for census users, since it will essentially produce population estimates for all local authorities simultaneously.

One of the typical key assumptions underpinning DSE type methods is that neither source has any over-coverage. The CCS is designed specifically to minimise this, but inevitably the Census will suffer from some especially as more modes of response have been made available. In terms of the overcoverage estimation, it is proposed to build on the 2011 approach, but also investigate if modelling of overcoverage in a similar way to the modelling of undercoverage could provide some gains in quality of the estimates.

The residual bias adjustments in the 2021 Census is planned to be broadly similar to the one used in the 2011 Census. For the household bias adjustment (which attempts to address bias due to the dependency between census and the CCS) the alternative household estimate is going to be produced based on the dummy forms and then incorporated into the undercoverage estimates [Brown *et al.,* 2006]. Options of using some administrative data to obtain the alternative estimates and combine them with the general estimates are also being considered. Certain challenges related to the use of administrative data exist. The main two challenges being data access and overcoverage. Some ways around the overcoverage may be considered, say, producing the weighting class weights as alternatives to the general estimation weights. These weights may be reasonably well protected against the overcoverage in the case of independence between the census response and administrative data overcoverage. Nevertheless, the independence is not necessarily guaranteed as, say, some of the census non-responding households may be non-occupied, but ‘populated’ with admin records. The ideal situation would be to have access to an administrative data source which has near to zero overcoverage – this would likely mean it also has high undercoverage which increases variance. However, at present we are not aware that such a source exists and is available to ONS, but efforts will continue to explore research in this area. Given that the alternative estimates usually have larger variances than the original estimates, the composite estimation approach is going to be considered for achieving a better balance between bias and variance.

In theory, an alternative to the household bias adjustment would be the multiple system estimation approach, for instance, the triple system estimation (TSE) approach. Even though the theory behind the TSE is well-developed, there are serious practical limitations preventing the national statistical offices from implementing it in the census context [Baffour *et al.,* 2013]. As it stands, the DSE was used for the census coverage estimation in the most recent censuses [Chipperfield *et al.,* 2017, Stats New Zealand, 2018] and is going to be used in the forthcoming ones [US Census Bureau, 2017]. Most of the data sources that may be regarded as candidates for a third list are administrative sources and same challenges as in the case of the bias adjustment exist: access and overcoverage. Moreover, the existing legislation limits implementation of various form of dependent interviewing that may be needed for the TSE with administrative data as a third list, since the TSE assumes absence of erroneous inclusions on all lists [Griffin, 2014]. Given the uncertainty around access to the relevant administrative data, development time and a need for a reliable and timely estimation methodology, the TSE approach was ruled out.

The method for the national adjustment using sex ratios will resemble the one used in 2011, although we expect additional sources to be used to define the target national sex ratios. The set of rules to invoke the national adjustment is yet to be prepared. In addition, the research on impact of the national adjustment on the subnational level estimates is being planned.

Coverage estimation of communal establishments is planned to be very similar to the one used in 2011, although discussions are ongoing about the scope of the CCS and the use of administrative data for assessing coverage of larger CEs.

1. **Work done so far**

The following work on the 2021 CCE has been completed so far.

* Analysis of the residual heterogeneity bias when the DSE is post-stratified by LA, HtC, area cluster and age-sex [Racinskij, 2017]. Work concluded that the relative residual bias could be at least -0.2% at the national level.
* Work on a data blended DSE (DBDSE) [Racinskij, 2018]. The DBDSE treats the administrative data records for the census non-responding households as real census records (hence the data blended). The research found no evidence of any substantial gains in using the DBDSE and showed a potential increase of uncertainty in the resulting population estimates.
* Work exploring the likely extent of the synthetic bias in the synthetic LA estimation.
* Exploring the use of continuous age variable in logistic regression DSE using the restricted cubic splines approach, to facilitate improving estimates of single year of age to address a lesson learnt from 2011. At the moment the scope of this work is only to enable more complex simulated census and CCS coverage patterns, but further research could be done to explore a single year of age population estimate.
* Optimization and parallelisation of the estimation system to explore the feasibility of running models for the entire country both in the research environment and in the real census processing platform. Achieved performance allows implementation of the proposed approach, work is used as a performance benchmark for the SAS server on which the 2021 CCE is going to be processed.
* Exploring the logistic and mixed effects logistic regression approach for the coverage modelling and population size estimation at the national and LA levels. Initial results show feasibility of achieving higher accuracy population estimates compared to the benchmark method, both in terms of bias and variance.
1. **Work in progress**

Several large projects within the 2021 CCE workstream are ongoing now:

* Work on bridging the gap between the CCS design and the proposed CCE strategy. Approaches on reflecting the CCS design in the undercoverage modelling are being investigated and work on the optimal design required for the coverage models is under way.
* Work on the model (and variable) selection procedures and goodness-of-fit assessment strategy has commenced.
* Work supporting the building of the SAS server for the 2019 Rehearsal and the 2021 Census.
1. **Planned work**

Below is a list of areas for further investigation. These areas were either identified during the current research or known from the 2011 CCE (High, Medium or Low priority in brackets)

* Work on making simulation studies more realistic (reduction of over-smoothing). This will be important to demonstrate how robust the methodology is against failure in assumptions as well as performance under difference levels of non-response. (High).
* Introducing the multiple modes into the simulation studies. (High).
* Operational test of the prototype of the estimation system in the 2019 Test. (High).
* Variance estimation (including non-symmetric confidence intervals. (High).
* Overcoverage estimation. (High).
* Household bias adjustment (including options with admin data). (High).
* National adjustment. (High).
* Contingency strategy. (High).
* Theoretical paper on coverage estimation. (High).
* Producing a set of estimates for a small number of LA using the 2011 Census and CCS data (rather than the simulated data) and the methods proposed for the 2021 CCE. This will help to explore (up to a point) the performance of the methods on non-smoothed data. (Medium).
* Work on modelling the 2011 Census and CCS data with inclusion of various area-level covariates such as census return rates, etc., to start exploring the usefulness of are-level covariates. (Medium).
* Getting the 2011 undercoverage estimation method ready as a backup option. (Medium).
* Sample balance work (if needed). (Medium).
* Exploration of the extent of total error in population size totals. (Medium).
* Aligning the research on coverage estimation with research on coverage adjustment. (Medium).
* Adjustment for linkage errors. (Low).
* A paper on coverage estimation for the census users. (Low).
1. **Summary**

This paper outlined the current state of the census coverage estimation workstream. The CCE work is well under way with several key projects completed. The remaining work is identified and the entire CCE package planned to be ready by the end of 2020.

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