Evaluating the DPM estimation method using a simulation study

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Executive summary

The Office for National Statistics (ONS) has been exploring methods for making better and more extensive use of administrative data to improve the accuracy and timeliness of population and migration statistics, including mid-year population statistics and components of change.

We used the method of Particle Filters (PF), see, for example Doucet and Johansen, (2008), to estimate the demographic account of England and Wales between 2011 and 2022 using data from a variety of sources, including estimates of migration statistics based on survey data, admin-based population statistics based on the Patient Register (PR) and SPDs (which are obtained by linking a number of administrative sources). The method performed reasonably well compared with the current method for MYEs but it is computationally intensive, see Office for National Statistics (2023). Furthermore, the effective sample size can be very low in some ages, which affected the validity of the credible intervals that were produced.

To address these limitations, we considered an alternative estimation method based on the Laplace approximation. It is implemented in a package called Template Model Builder (TMB); see Kristensen et al. (2016). It is much faster than PF method - only 20 minutes are needed to process all local authorities instead of 15 hours for PF.

In this paper, we consider a detailed evaluation of the new method, we refer to as the TMB, using a simulation study. We first evaluate TMB when all the input data to the method are free of systematic bias – we refer to this as the benchmark scenario. We then evaluate TMB when systematic errors are introduced to parts of the input.

We describe briefly the TMB estimation method and the design of the simulation. We present results based on data from two local authorities: Blackpool and Cambridge. The former displays modest flow rates whereas the latter displays very large inflow rates at age 18 (university entry age).

To evaluate the method, we use mostly estimates of relative bias and the observed coverage and width of 95% credible intervals.

We found that under the "benchmark" scenario the method showed very low levels bias for population estimates and generally modest positive bias for estimates of inflows and outflows. The exceptions are in:

- ages 18 and 19 when inflow rates are very high, which occurs in university towns/cities such as Cambridge.
- old ages, where inflow and outflow rates are very low, which is seen in most local authorities.

In addition to the simulation study, we have run TMB using real data (data used in recent publications) but without the use of census 2021 data and compared the estimates obtained

with the published census 2021 rebased 2021 MYEs. An analysis of the results shows that the DPM estimates drift less other time than in the current MYE method.

Key asks of MARP

1. Comment on the design of the simulation study and our evaluation of the method
2. Recommend ways to explore further the limitations of the method.

1. Introduction

We initially used the PF method to estimate the demographic account of England and Wales between 2011 and 2022 using data from a variety of sources. The method performed reasonably well compared with the current method for MYEs but it is computationally intensive and the effective sample size was found to be very low in some ages, which affected the validity of the credible intervals that were produced.

To address these limitations, we considered an alternative estimation method based on the Laplace approximation, implemented in a package called TMB.

In this paper, we consider a detailed evaluation of the new method, we refer to as the TMB method, using a simulation study. We first evaluate TMB when all the input data to the method are free of systematic bias – we refer to this as the benchmark test. We then evaluate TMB when systematic errors are introduced to parts of the input.

We describe briefly the TMB estimation method and the design of the simulation. We present results based on data from two local authorities: Blackpool and Cambridge. The former displays modest flow rates whereas the latter displays very high inflow rates at age 18 (university entry age).

To evaluate the method, we use estimates of relative bias, the observed coverage of 95% credible intervals and the root mean square error (RMSE).

We also present results of a run of the DPM using real data but without 2021 census data to look at the potential drift in DPM estimates and compare it with that in the current MYE method.

2. Brief description of the estimation method

The DPM estimates separately the demographic account of each cohort between times 0 and T (a cohort is composed of individuals of a given sex born in the same year ending in June). This involves producing estimates of the population and components of change (births, deaths, inflows and outflows) by local authority, single year of age (SYOA) and sex at the end of June of each year, together with measures of uncertainty.

Let c, a and t denote a cohort, age and time, respectively.

Note: for simplicity, we do not include a subscript for sex in the notation below.

Let $\gamma_{a,t+1}^{bth}$, $\gamma_{a,t+1}^{dth}$, $\gamma_{a,t+1}^{in}$ and $\gamma_{a,t+1}^{out}$ denote the rates of births, deaths, inflows and outflows, respectively, at age a between times t and $t+1$.

We assume that the demographic rates have Gamma distributions with means

$\mu_{a,t+1}^{bth}$, $\mu_{a,t+1}^{dth}$, $\mu_{a,t+1}^{in}$ and $\mu_{a,t+1}^{out}$ and dispersion parameters δ^{bth}, δ^{dth}, δ^{in} and δ^{out}. We assume that the dispersion parameter is constant within a local authority, but this assumption can be relaxed.

The mean rates are initially estimated by fitting Generalised Additive Models (GAMs) to estimates of births, deaths, inflows and outflows.

The dispersion parameters are derived from the variances of "raw" rates (calculated from estimates of flows and a population base).

Let y_t^{stk}, y_t^{in} and y_t^{out} be vectors that denote the available data on stocks, inflows and outflows at time t – some of the components may have missing values (data may not be available for some ages).

Data, especially from administrative sources, can suffer from bias. A coverage adjustment in the form of a coverage ratio needs to be estimated.

For stocks, the coverage ratios are obtained by either fitting models to data from a coverage survey or models of the ratio of observed counts from a data source around the time of the census to census-based estimates (eg, census 2021-based mid-year estimates).

Let $x_{c,0,T}$ be a vector that denotes the unobserved demographic account of cohort c between times 0 and T. Its components represent population stocks over time and components of change in the Lexis triangles between times 0 and T (see below).

Let $x_{c,a,t}^{stk}$ denote the population stock of cohort c at time t, when age is a.

Let $x_{c,a,t+1}^{dth}$, $x_{c,a,t+1}^{bth}$, $x_{c,a,t+1}^{in}$ and $x_{c,a,t+1}^{out}$ denote the number of deaths, inflows and outflows, respectively, between times t and $t+1$.

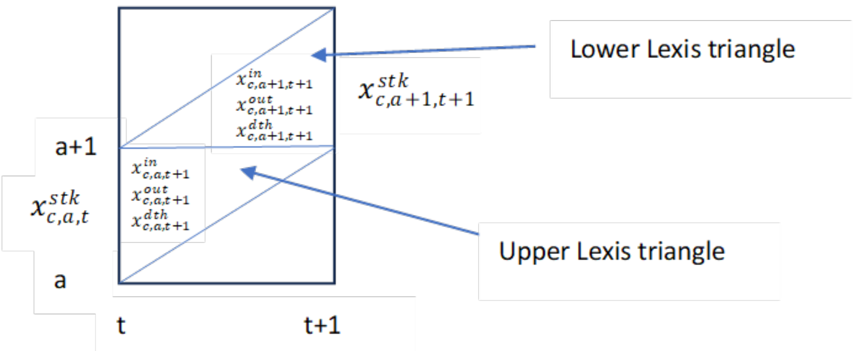
Let $x_{c,a+1,t}^{stk}$ denote the number of individuals in cohort c who reach age $a+1$ before time $t+1$ (this stock is referred to as accession).

Figure 1 shows Lexis triangles between times t and $t+1$.

The demographic accounting identity dictates that

$$x_{c,a+1,t}^{stk} = x_{c,a,t}^{stk} + x_{c,a,t+1}^{in} - x_{c,a,t+1}^{dth} - x_{c,a,t+1}^{out}.$$

Figure 1. Lexis triangles of a cohort between two successive time points



The objective is to estimate the joint posterior distribution of the account $x_{c,0,T}$ given the available data, their coverage ratios, which we denote by ρ, and the initial estimates of mean rates and dispersion parameter values, which we denote by μ and δ respectively.

The estimation is achieved by assuming models for the unobserved components of change and models for the data. The former represent regularities in demographic processes and are referred to as system models. The latter represent systematic errors and uncertainty of the data and are referred to as data models.

A more detailed description of system models and data models can be found in Annex 1.

Estimating the posterior distribution

Because of the demographic accounting identity, the problem can be reduced to the estimation of the joint distribution of the population at time 0 and the inflows and outflows along the Lexis triangles between times 0 and T. The distribution of the population is derived by applying the demographic accounting identity to a sample from the estimated posterior distribution.

Let

$p\left(x_{c,a,0}^{stk}, x_{c,a,1}^{in}, x_{c,a,1}^{out}, \dots, x_{c,A,T}^{in}, x_{c,A,T}^{out} | \mathrm{y}_{0,T}^{stk}, \mathrm{y}_{1,T}^{in}, \mathrm{y}_{1,T}^{out}, \mathrm{x}_{c,1,T}^{bth}, \mathrm{x}_{c,1,T}^{dth}; \mu, \delta, \rho\right)$ denote this joint posterior distribution.

Births and deaths from registrations are assumed to be known and exact.

Applying Bayes theorem allows us to write the posterior distribution as a product of likelihoods based on the data models, distributions of the unobserved account based on the system models, prior distributions of some parameters (eg, population stock at time 0) and a normalising constant.

We use the Laplace approximation method, which yields a posterior distribution that is a multivariate normal. To ensure that the values of the account are non-negative, we express the un-normalised posterior in terms of the logarithm of each unobserved component – we include appropriate Jacobian factors to account for the transformation.

For inflows, $x_{c,a,t}^{in} = e^{z_{c,a,t}^{in}}$, where $z_{c,a,t}^{in} = \log\left(x_{c,a,t}^{in}\right)$.

We do the same for outflows and population stocks at time 0.

Applying the Laplace approximation to the transformed un-normalised posterior leads to the vector $\left(z_{c,a,0}^{stk}, z_{c,a,1}^{in}, z_{c,a,1}^{out}, \dots, z_{c,A,T}^{in}, z_{c,A,T}^{out}\right)$ to follow a multivariate normal distribution.

TMB is used to implement the Laplace approximation method and compute the mean and variance-covariance matrix of the multivariate normal distribution.

A sample of size N is drawn from the distribution (usually N is set to 1000); the values are then exponentiated to yield a sample of values of the initial population and the inflows and outflows between times 0 and T. The demographic accounting identity is then used to derive the population stocks after time 0 on each element of the sample.

Posterior distributions for the rates can also be obtained.

95% credible intervals for estimates of the population, inflows and outflows are obtained from the sampled values.

Additional parameters, which we refer to as scale parameters, to correct for potential bias in the coverage ratios, can also be added to the model.

3. Description of the design of the simulation study

The design of the simulation study involves: (i) generating a “true” account from specified demographic mean rates and starting population values; (ii) generating data for stocks, inflows and outflows according to the data generating mechanism; (iii) running the model with specified parameters.

(i) Generating a “true” account

We simulate a “true” account using the 2011 MYEs as the starting stocks and rates that are draws from a Gamma distribution with a mean equal to the smoothed rates from the December 2023 publication and specified dispersion values.

For each draw of rates, we simulate the true account separately for each cohort and sex across time from 2011 to 2023, moving along the Lexis triangles.

We describe below the algorithm for generating the true inflows, outflows and deaths in the upper Lexis triangle between times t and $t+1$ for run r. It is based on the transition function of the demographic account.

Step 1. We simulate “true” demographic rates, $\gamma_{a,t+1}^{in,(r)}$, $\gamma_{a,t+1}^{out,(r)}$ and $\gamma_{a,t+1}^{dth,(r)}$, by drawing from Gamma distributions with the specified parameters (see above).

Step 2. We simulate a count, $x_{c,a,t+1}^{in}$, for inflows by generating a random number from $Pois(\gamma_{a,t+1}^{in,(r)}/2)$ (we divide by 2 as we simulate a count for a triangle).

Step 3. We simulate joint counts, $x_{c,a,t+1}^{out}$ and $x_{c,a,t+1}^{dth}$, for outflows and deaths, respectively by generating a pair of random numbers from a multinomial distribution $multinom(Size, P_{c,a,t+1}^{out}, P_{c,a,t+1}^{dth}, P_{c,a,t+1}^{remain})$ – the parameters of the multinomial can be found in Annex 2

- This method uses the competing risks formulation (see Preston et al. (2001)) and is an approximation that uses discrete time – it assumes that half the inflows arrive at the start of the period and half at the end of the period.

Step 4. Accession (the number of individuals in the cohort who reach age a+1 before time t+1) is set equal to $x_{c,a,t}^{stk} + x_{c,a,t+1}^{in} - x_{c,a,t+1}^{dth} - x_{c,a,t+1}^{out}$.

For simplicity, we haven’t included the simulation of the number of births in the above algorithm.

We have run some checks to verify that the algorithm worked; see Figure A1 in Annex 2

(ii) Simulating data: stocks, inflows and outflows

We consider a number of scenarios: a “benchmark” scenario, where all input is free of systematic bias, and scenarios where we introduce errors by changing the coverage ratios of the stocks or flows data by a specified amount.

Let $\tilde{\rho}_{a,t} = (1 + rb^{stk})$ denote the coverage ratio used in the simulation of stocks data.

Stocks are drawn from $NB(\tilde{\rho}_{a,t} x_{c,a,t}^{stk}, 0.0001)$ (approximately a Poisson distribution).

Inflows and outflows are drawn from $NB\left((1 + rb^{in}) x_{c,a,t+1}^{in}, 0.05\right)$ and $NB\left((1 + rb^{out}) x_{c,a,t+1}^{out}, 0.05\right)$, respectively.

(iii) Running the model

The distributions in the data models used to run the model match the distributions used to generate the data; that is, we use the negative binomial, and the same values for the dispersion parameters.

The coverage ratios in the data models are set to 1. Thus, when $rb^{stk} \neq 0$, the assumed coverage ratio of stocks data is biased.

Let $\tilde{\mu}^{in}_{c,a,t+1} = \left(1 + rb^{in}\right)\mu^{in}_{c,a,t+1}$, and $\tilde{\mu}^{out}_{c,a,t+1} = \left(1 + rb^{in}\right)\mu^{out}_{c,a,t+1}$ denote the mean rates used in the system models.

Similarly, when $rb^{in} \neq 0$ and or $rb^{out} \neq 0$, the input for flows are biased.

The mean rates for births and deaths are set equal to the “true” mean rates.

In the benchmark scenario, the mean rates of all the system models match the “true” mean rates.

The values of the dispersion parameters used in the system models match the values used to generate the “true” rates in all scenarios.

For the benchmark scenario, the model was run 500 times, whereas for the other scenarios the model was run 250 times.

Question 1: is the design of the simulation adequate?

4. Simulation results

4.1 Criteria for evaluation

To evaluate the performance of TMB, we use the relative bias of the estimates (mean of the posterior distribution) and the observed coverage of the 95% credible intervals as the main criteria.

For some comparisons, we use also use the relative width of the credible intervals and the root mean square error (RMSE).

To estimate the relative bias of an estimate, we compute the percentage difference between the mean of the distribution and the “true” value for each run, and then average over all runs.

The observed coverage of the 95% credible interval is given by the percentage of runs where the true value is included in the credible interval.

4.2 Evaluation of the benchmark scenario

In this scenario, where there is no systematic bias in the model parameters, we expect the relative bias to be around 0, with approximately the same number of estimates below and above 0 and observed coverage of around 95%.

We describe the simulation results obtained using data for Blackpool and Cambridge. We present plots for 2017, which is the middle of the period between 2011 and 2023. The results for other years are similar.

Figure 2 shows the performance of TMB for inflows into Blackpool. We can see that the relative bias tends to be positive but below 5% for ages below 75. The observed coverage in this age group is close to 95%, which indicates that the level of bias is modest and its impact on the validity of inference should not be large. On the other hand, for ages above 75, the relative bias can be high, especially in males, which can be an artifact of low inflow rates. In males, the very high levels of relative bias seem to have a big impact on the observed coverage (values way below 90%).

Estimates of outflows show the same pattern. Because inflows and outflows tend to be biased in the same direction, the impact of the bias on population estimates is small (see Figures A2 and A3 in Annex 3).

Figure 2. Relative bias and observed coverage of inflows into Blackpool

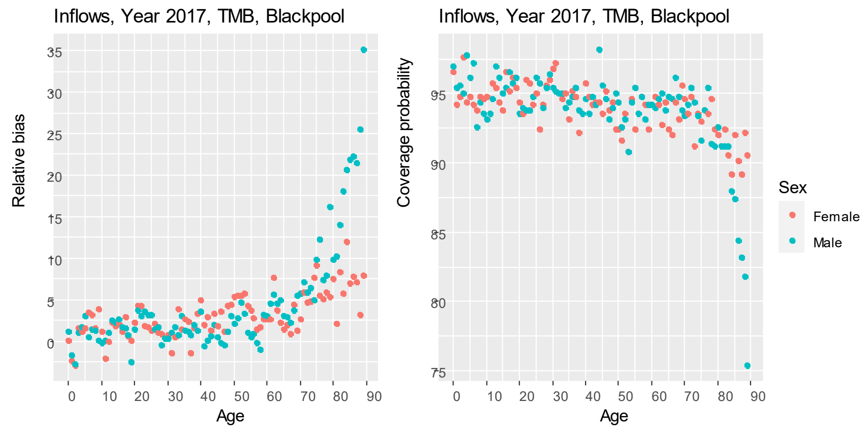


Figure 3 shows the relative bias and observed coverage of inflows into Cambridge. Similarly to Blackpool, the relative bias tends to be positive in ages below 75, but we see some values close to -10% (for age 19). However, the observed coverage does not seem to be affected much (it is around 92%). We also see the same problem as Blackpool for old ages.

For outflows, the performance using Cambridge data is slightly different from that of Blackpool. As we can see from Figure 4, the relative bias at age 18 is nearly -20% and at age 19 it is below -10%. The impact of this level of bias can be seen clearly in the observed coverage which can be as low as 75% for age 18 for both males and females.
The mean inflow rates at ages 18 and 19 in Cambridge are much higher than in Blackpool. Results using data from Oxford, which is also a university town, show the same pattern.
It is unclear why TMB seems to struggle to estimate cohorts where there is a high spike in mean inflow rates.

We have run the simulation using PF; the results do not show the same pattern as TMB (see Figure A4 in Annex 3). The relative bias tends to be distributed symmetrically around 0 across all ages. However, the observed coverage values tend to be below 95%, which indicates that the credible intervals are misleadingly too narrow.

As the bias in inflows and outflows is in the same direction, the impact on population estimates is small, except for very old ages, as can be seen in Figure 5.

Figure 3. Relative bias and observed coverage of inflows into Cambridge (TMB)

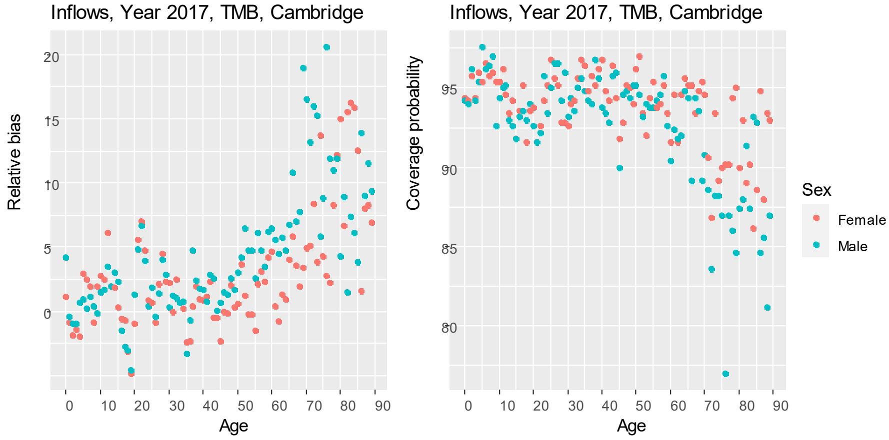


Figure 4. Relative bias and observed coverage of outflows into Cambridge (TMB)

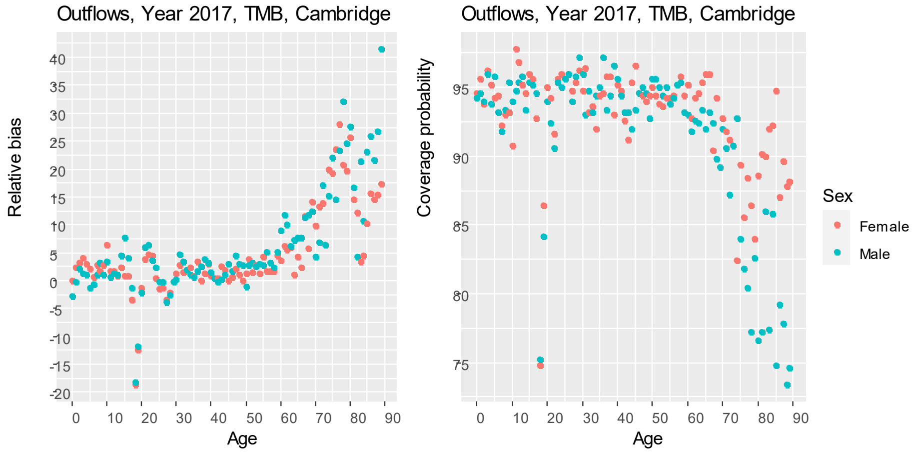
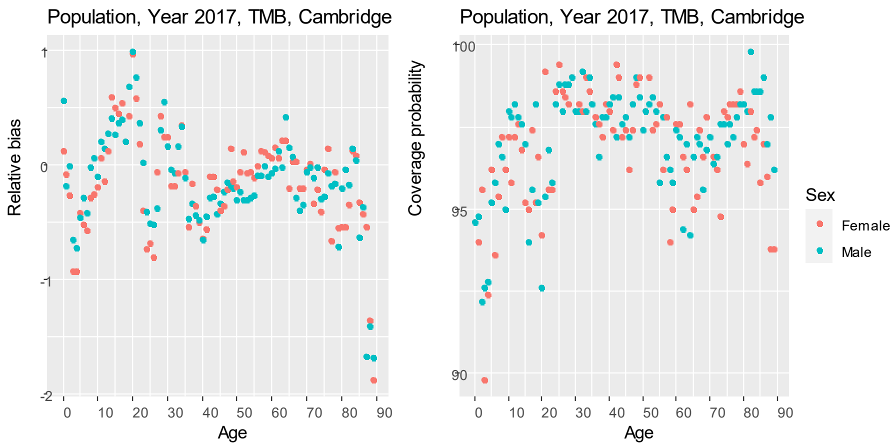


Figure 5. Relative bias and observed coverage of population estimates of Cambridge (TMB)



Question 2: do you have any suggestions on the source of the observed positive bias in flow estimates, and the large underestimation of outflows at ages 18 and 19 in Cambridge?

4.2 Performance of TMB under input errors

We have evaluated TMB under input errors: (i) biased flow data and unbiased stocks; (ii) unbiased flow data and biased stocks. For simplicity, the relative bias was held constant over time and age. The results show that:

- bias in part of the input leads to bias in both flows and population estimates
- population estimates are more affected by bias in population stocks
- the scale parameter for coverage ratios reduces the level of bias

For more details, see Annex 4.

Question 3: are there any other scenarios that you would suggest for evaluation?

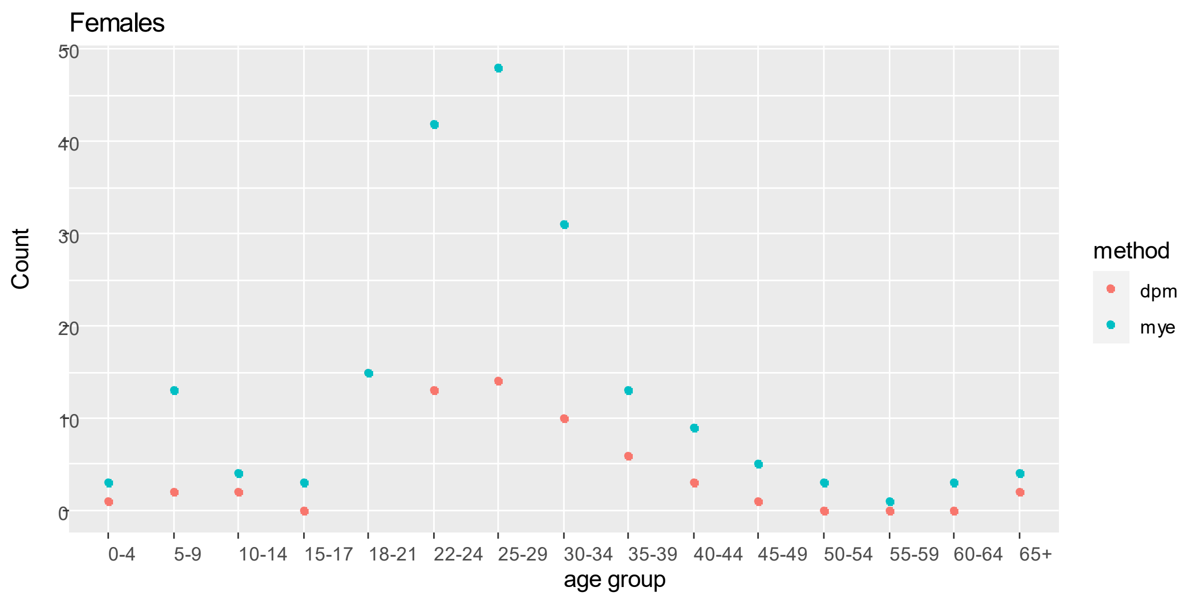
5. Further evaluation of TMB using real data without census 2021 data

To understand the potential drift in DPM estimates that could result from estimating coverage ratios using past census data, we have run the model for the period 2011 to 2021 using, 2011 MYEs, PR (for 2012-2015) and SPDv4.2 (for 2016-2021) as stocks data – the admin data were coverage adjusted using 2011 MYEs, and hence no census 2021 data were used in the estimation. The migration data are the same as that used in the current MYE publication. We then compared the 2021 DPM estimates and rolled forward MYEs with the census 2021 rebased 2021 MYEs.

This comparison shows that DPM produces fewer local authorities with extreme percentage differences from census 2021 rebased 2021 MYEs (more than 15%) than rolled forward MYEs (Figure 6 shows this for estimates of females by age group). Other plots can be found in Annex 5.

We expect DPM to perform better if a more robust coverage adjustment for SPDs was used.

Figure 6. Number of local authorities with extreme differences from 2021 MYEs in females



As a coverage adjustment method for SPDs is still in development, we are currently using coverage ratios based on census 2021 for years after 2021 in the DPM publication.

Question 4: are the results obtained for the 2011-2021 period reassuring with regard to our approach?

6. Future work

We are aiming to use MCMC methods to estimate the posterior distribution without using the Laplace approximation to understand the source of the observed bias. MCMC would not be practical in production but should be useful for investigation work. We are considering using importance sampling to correct for the use of the Laplace approximation.

For old ages, where TMB clearly underperforms, we will explore using the “extinct generation” method (Andreev et al. (2002). It is based on the assumption that the population drops to 0 after a certain age; working backwards then allows the production of estimates for ages up to the starting period.

Question 5: are there other methods that are worth pursuing?

References

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Annex 1 System models and data models

System models

We describe the system model of outflows in the upper Lexis triangle as an example. It is given by:

$$x_{c,a,t+1}^{out} \sim Pois\left(\gamma_{a,t+1}^{out} e_{c,a,t+1}\right),$$

where $\gamma_{a,t+1}^{out} \sim Gamma\left(\frac{1}{\delta}, \frac{\delta}{\mu_{a,t+1}^{out}}\right)$

and

$e_{c,a,t+1} = \frac{\left(x_{c,a,t}^{stk} + x_{c,a+1,t}^{stk}\right)}{4}$.

The term $e_{c,a,t+1}$ is called "exposure" and refers to the population base that could emigrate in the upper Lexis triangle between times t and $t + 1$.

This exposure applies also to deaths and births; for inflows, the exposure is equal to 1.

Data models

We describe the data model of stocks as an example. Let $y_{a,t}^{stk}$ denote the population stock of age a at time t. The data model is given by

$$y_{a,t}^{stk} \sim N\left(\rho_{a,t}^{stk} x_{c,a,t}^{stk}, {\sigma_{a,t}^2}^{stk}\right),$$

Where $\rho_{a,t}^{stk}$ denotes the coverage ratio of the population stock and ${\sigma_{a,t}^2}^{stk}$ the variance of the distribution.

Other distributions can be used in data models, including the negative binomial.

The coverage ratios are obtained from either models of data from a coverage survey or models of the ratio of observed counts from a data source around the time of the census with census-based estimates (eg, census 2021-based mid-year estimates).

A parameter $v_{a,t}^{stk}$ for adjusting for potential bias in the estimates of coverage ratios can be added. Its prior distribution is normal with mean 0 and standard deviation $s_{a,t}^{stk}$, which we refer to as the coverage ratio scale parameter.

The data model then becomes:

$$y_{a,t}^{stk} \sim N\left(e^{v_{a,t}^{stk}} \rho_{a,t}^{stk} x_{c,a,t}^{stk}, {\sigma_{a,t}^2}^{stk}\right).$$

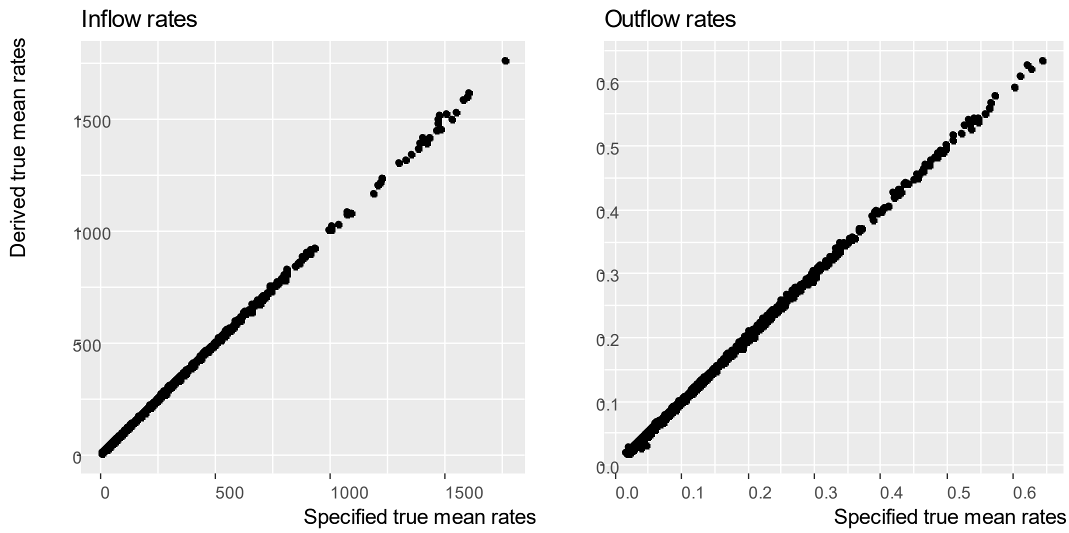
Annex 2 Simulating a true account

Once inflows are drawn, we draw outflows and deaths from a multinomial distribution with the following parameters:

- $Size = x^{stk}_{c,a,t} + x^{in}_{c,a,t+1}/2$
- $P^{out}_{c,a,t+1} = \frac{\gamma^{out,(r)}_{a,t+1}}{\gamma^{dth,(r)}_{a,t+1}+\gamma^{out,(r)}_{a,t+1}}\left(1 - e^{-\frac{1}{2}\left(\gamma^{dth,(r)}_{a,t+1}+\gamma^{out,(r)}_{a,t+1}\right)}\right)$
- $P^{dth}_{c,a,t+1} = \frac{\gamma^{dth,(r)}_{a,t+1}}{\gamma^{dth,(r)}_{a,t+1}+\gamma^{out,(r)}_{a,t+1}}\left(1 - e^{-\frac{1}{2}\left(\gamma^{dth,(r)}_{a,t+1}+\gamma^{out,(r)}_{a,t+1}\right)}\right)$
- $P^{remain}_{c,a,t+1} = e^{-\frac{1}{2}\left(\gamma^{dth,(r)}_{a,t+1}+\gamma^{out,(r)}_{a,t+1}\right)}$

To verify that the algorithm we have used generates accounts that are consistent with the specified mean rates, we computed the rates from the data of the simulated account from each run for every Lexis triangle and averaged the rates over all runs. We then compared these rates, which we refer to as derived mean rates, with the specified mean rates. Figure A1 shows that the derived mean rates match the specified mean rates very well for both inflows and outflows.

Figure A1. Comparing mean rates based on simulated true account data and specified mean rates



Annex 3 Additional simulation plots for the benchmark scenario

Figure A2. Relative bias and observed coverage of outflows (Blackpool)

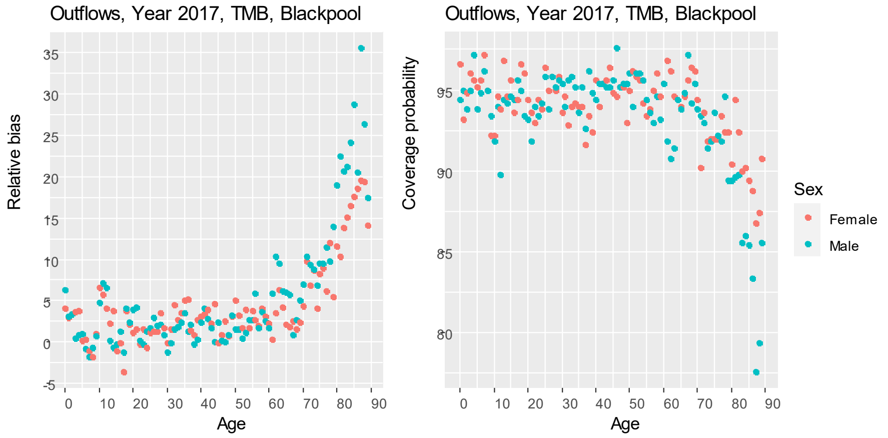
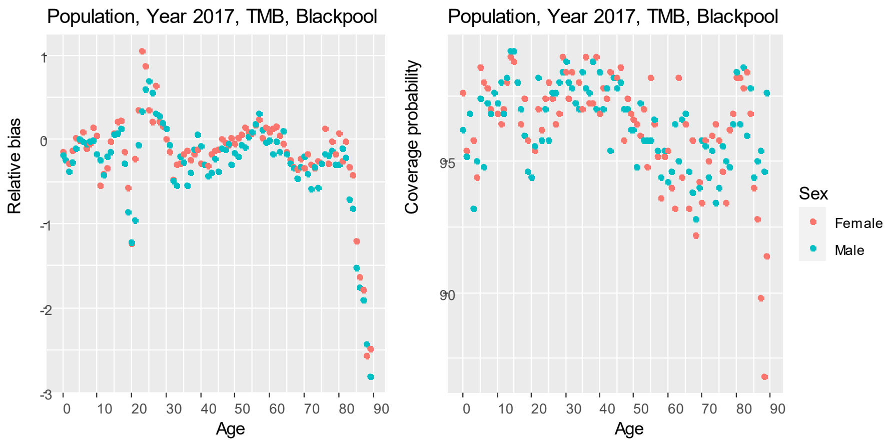
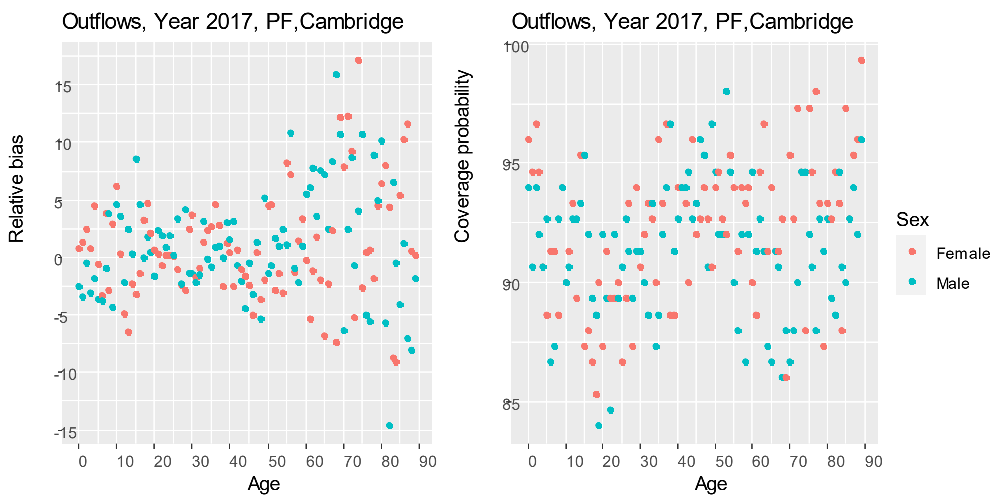


Figure A3. Relative bias and observed coverage of population estimates (Blackpool)



Performance of the Particle Filter

Figure A4. Relative bias and observed coverage of outflows into Cambridge (PF)



Annex 4: Evaluation of TMB under input error

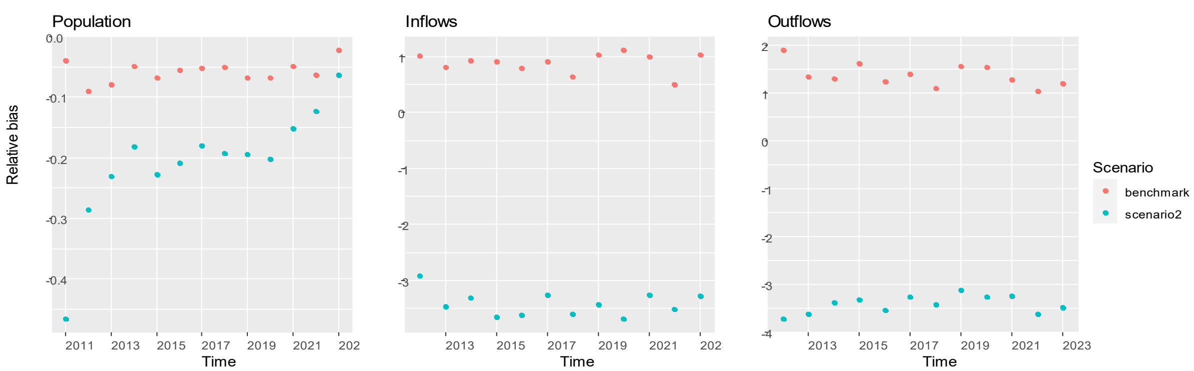
A4.1 Errors in flow rates and coverage ratios of flows

In this scenario (which re refer to as scenario 2), the mean rates of flows are set below the true rates by 10% ($rb^{in} = rb^{out} = -0.1$) . The coverage ratio of the simulated flow data is 0.9, that is 10% below the ratio used in the data models.

The coverage ratio of the simulated stocks data is equal to the ratio used in the data models.

As we can see in Figure A5, which shows the relative bias of the estimates at the local authority level, the difference between the levels of relative bias of the benchmark scenario and scenario 2 is around 5 percentage points. The impact on population estimates is on the other hand small. This indicates that having unbiased stocks protects against bias in flow data in an important way.

Figure A5. Impact of biases mean rates on LA level estimates - Cambridge



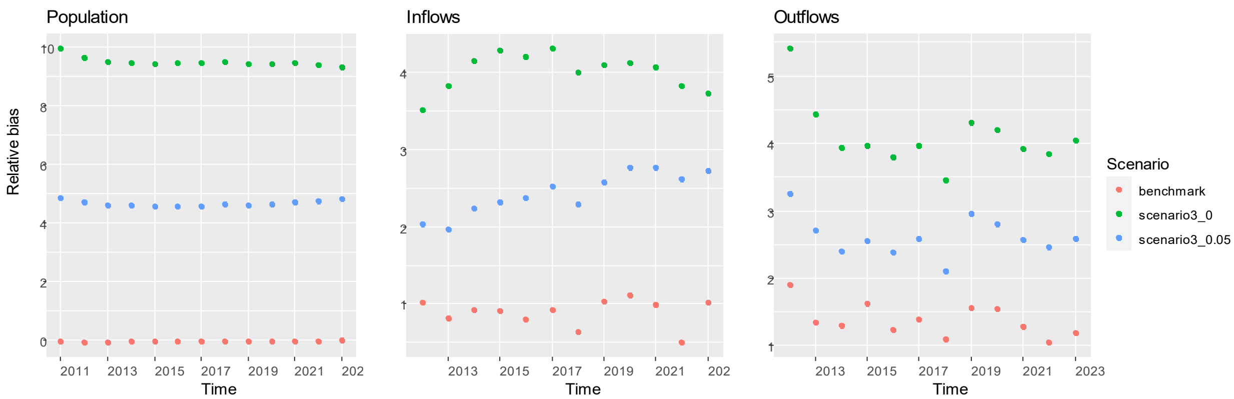
A4.2 Errors in coverage ratios of stocks

Here, the flow parameters are unbiased but we introduce a bias into the coverage ratios of stocks $\left(rb^{stk} = 0.1\right)$. We have run the simulation with and without the scale parameter for coverage ratios; the latter is referred to as scenario_0 whereas the former is referred to as scenario3_0.05 (the scale ratio was set to 0.05).

As we can see in the left panel of Figure A6, when the scale parameter is not used, the relative bias for population is close to 10%. However, using the scale parameter, which allows for bias correction, leads to a reduction of the relative bias. Setting the scale parameter to 0.05 reduces the relative bias to around 5%. Increasing the scale parameter reduces further the relative bias but increases the width of the credible intervals and RMSE values.

Bias is the coverage ratio of stocks leads to bias in the estimates of inflows and outflows. The middle and right panels of Figure A6 show that the relative bias increased by less than 2 percentage points when the scale parameter is used and by about 3 percentage points when no scale parameter is used.

Figure A6. – Impact of systematic errors in the coverage ratio of stocks



Annex 5: Running DPM without 2021 census data

Figure A7. Drift by sex

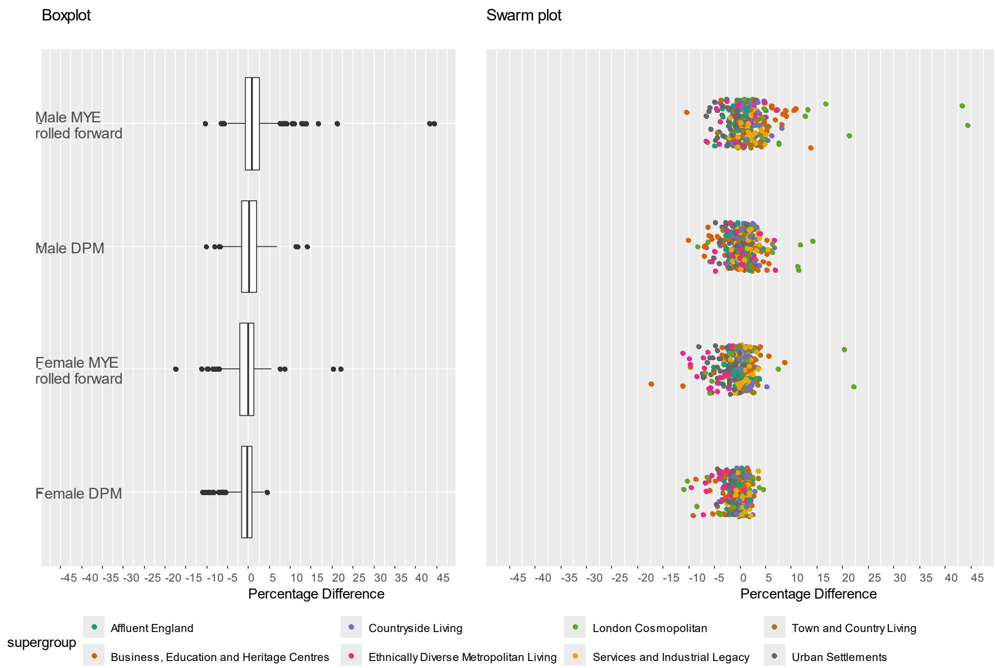
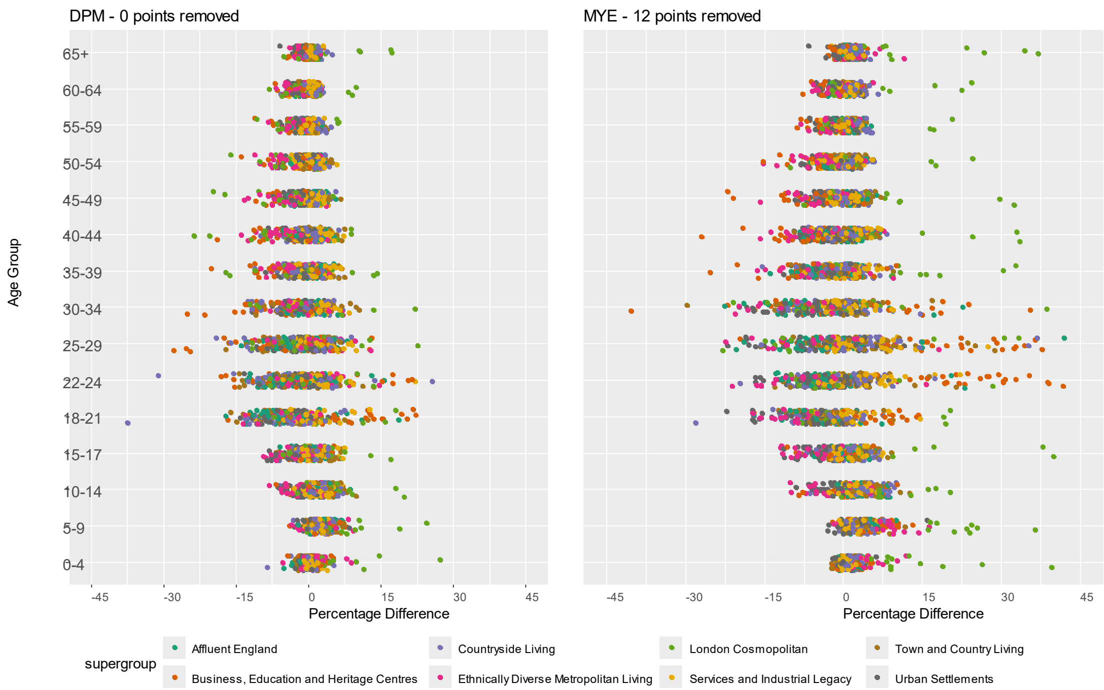


Figure A8. Drift by age group



Note: in Figure A8 we removed 12 points in the MYE plots for presentation purposes as they were very extreme.

Figure A9. Number of local authorities with extreme differences from 2021 MYEs in males